Answer on Question #74406 – Math – Calculus

Question

Check whether there exists a continuously differentiable function g defined by $f(x,y)=0$ in the neighbourhood of $x=3$,such that $g(3)=1/3$. Find $g'(3)$, if it exists.

Solution

Let

$$y=g(x)\,;\ f(x,y)=g(x)-y$$

If the limit

$$\lim_{h\to 0}\frac{g(c+h)-g(c)}{h}=g'(c)$$

exists for every $c\in(a,b)$, then we say that $g(x)$ is differentiable on (a,b).

Let

$$g(x)=\frac{1}{x}\ ;x\neq 0$$

Then:

$$\lim_{h\to 0}\left(\frac{\frac{1}{c+h}-\frac{1}{c}}{h}\right)=\lim_{h\to 0}\left(\frac{c-c-h}{hc(c+h)}\right)=\lim_{h\to 0}\left(-\frac{1}{c(c+h)}\right)=-\frac{1}{c^2}$$

Since the limit exists, the function $g(x)=1/x$ is differentiable function in the neighborhood of $x=3$ and there exists a continuously differentiable function g defined by $f(x,y)=0$ in the neighborhood of $x=3$ such that $g(3)\ =1/3$:

$$g'(x)=-\frac{1}{x^2}$$

$$g'(3)=-\frac{1}{3^2}=-\frac{1}{9}$$