

Question #74266

Calculate the volume of a parallelepiped whose sides are given by the vectors $a = 3\hat{i} + 2\hat{j} + k\hat{}$, $b = -\hat{i} + 3\hat{j}$ and $c = 2\hat{i} + 2\hat{j} + 5k\hat{}$

Answer:

The volume of a parallelepiped is equal to the absolute value of the determinant of a three dimensional matrix, which is built using vector defining the parallelepiped:

$$\begin{aligned} V &= \left| \det \begin{bmatrix} a_i & a_j & a_k \\ b_i & b_j & b_k \\ c_i & c_j & c_k \end{bmatrix} \right| = \left| \det \begin{bmatrix} 3 & 2 & 1 \\ -1 & 3 & 0 \\ 2 & 2 & 5 \end{bmatrix} \right| = \left| \det \begin{bmatrix} -1 & 3 \\ 2 & 2 \end{bmatrix} + 5 \cdot \det \begin{bmatrix} 3 & 2 \\ -1 & 3 \end{bmatrix} \right| = \\ &= |-2 - 2 \cdot 3 + 5(3 \cdot 3 + 2)| = 47 \text{ cubic units.} \end{aligned}$$

Answer provided by <https://www.AssignmentExpert.com>