Answer on Question #74094 – Math – Calculus

Question

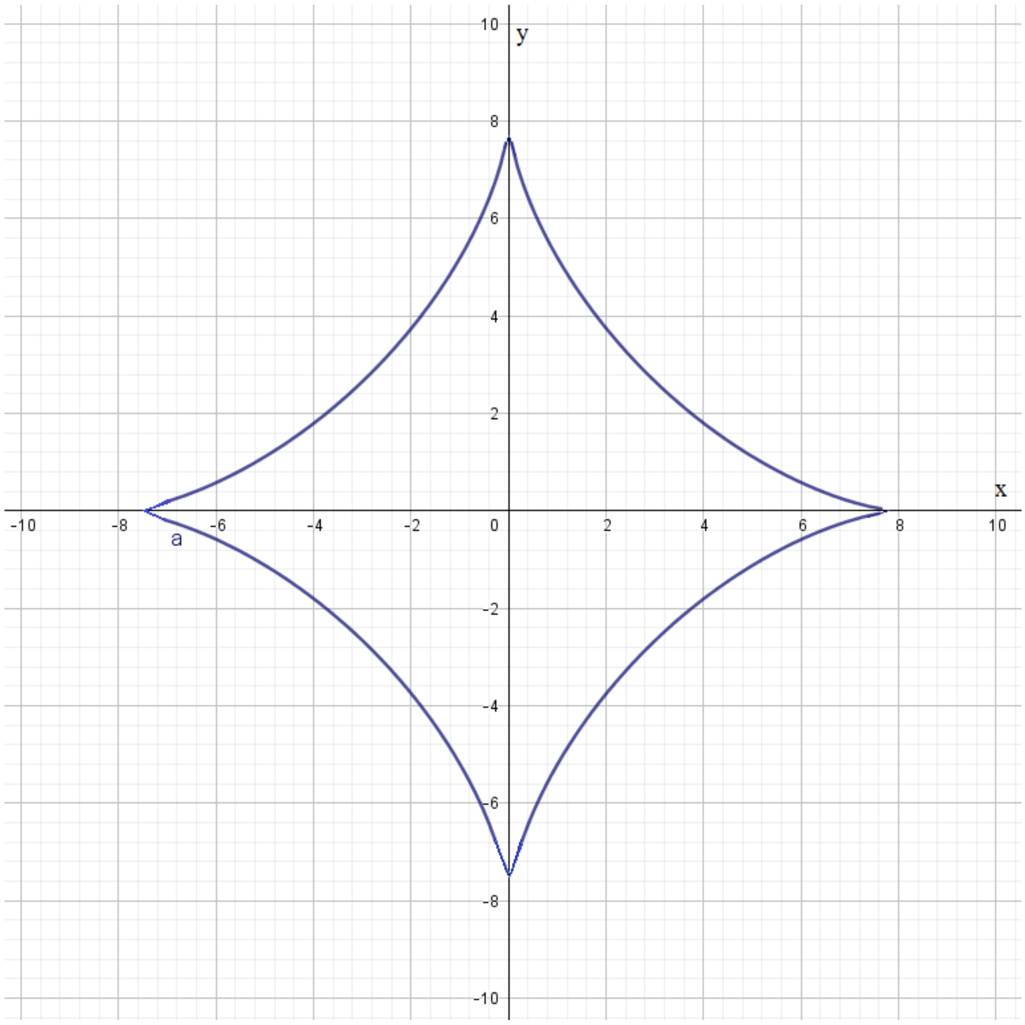
Find the volume of a solid obtained by revolving the curve

$$x = a\cos^3\theta\,, y = a\sin^3\theta$$

about the y −axis.

Solution

$$x = a\cos^3\theta\,, y = a\sin^3\theta\,, (0 \le \theta \le 2\pi)$$



The curve is called astroid.
When θ is changing from 0 to $\pi/2$, x is decreasing from a to 0.
The curve is closed and symmetric about the axes.
Then

$$V_y = 2\pi \int_{t1}^{t2} x(t)y(t)dx(t) = -\,2(2\pi)\int_0^{\pi/2} a\sin^3\theta\, a\cos^3\theta\, d(a\cos^3\theta) =$$

$$= 12\pi a^3 \int_0^{\pi/2} \sin^4\theta\, a\cos^5\theta\, d\theta$$

$$\int \sin^4\theta\, a\cos^5\theta\, d\theta$$

Substitution

$u = \sin\theta\,, du = \cos\theta\, d\theta$

$$\int \sin^4\theta\, a \cos^5\theta\, d\theta = \int u^4(1-u^2)^2\, du = \int (u^4 - 2u^6 + u^8)\, du =$$

$$= \frac{u^5}{5} - \frac{2u^7}{7} + \frac{u^9}{9} + C = \frac{1}{5}\sin^5\theta - \frac{2}{7}\sin^7\theta + \frac{1}{9}\sin^9\theta + C$$

$$V_y = 12\pi a^3 \left[\frac{1}{5}\sin^5\theta - \frac{2}{7}\sin^7\theta + \frac{1}{9}\sin^9\theta\right]\begin{matrix}\pi/2\\0\end{matrix} =$$

$$= 12\pi a^3 \left(\frac{1}{5} - \frac{2}{7} + \frac{1}{9} - 0\right) = \frac{32\pi a^3}{35}$$

Answer: $\frac{32\pi a^3}{35}$.