

Answer on Question #74078 – Math – Calculus

Question

The function f , defined by $f(x)=x^3-6x^2+16x-15$ is increasing in \mathbb{R} or not.

Solution

We are given the function

$$f(x) = x^3 - 6x^2 + 16x - 15$$

In order to find out whether the function is increasing or not, we use the Increasing/Decreasing Test:

(a) If $f'(x) > 0$ on an interval, then f is increasing on that interval.

(b) If $f'(x) < 0$ on an interval, then f is decreasing on that interval.

Find derivative of this function

$$f'(x) = (x^3 - 6x^2 + 16x - 15)' = 3x^2 - 12x + 16$$

To know where $f'(x) > 0$ and where $f'(x) < 0$ we solve the equation

$$f'(x) = 3x^2 - 12x + 16 = 0$$

Find discriminant

$$D = (12)^2 - 4 \cdot 3 \cdot 16 = 144 - 192 = -48 < 0$$

Since $D < 0$ the equation has no roots and derivative $f'(x)$ in \mathbb{R} does not change the sign. We define the sign of the derivative substituting for x any number, for example, zero:

$$f'(0) = 3 \cdot 0 - 12 \cdot 0 + 16 = 16 > 0$$

Therefore $f'(x) > 0$ for any x and $f(x)$ is increasing in \mathbb{R} .

Answer: The function $f(x) = x^3 - 6x^2 + 16x - 15$ is increasing in \mathbb{R} .