Answer on Question #74058 – Math – Differential Equations

Question

Do the functions $y_1(t) = \sqrt{t}$ and $y_2(t) = 1/t$ form a fundamental set of solutions of the equation $2t^2y'' + 3ty' - y = 0$, on the interval $t \in (,\infty)$?

Justify your answer.

Solution

$2t^2y'' + 3ty' - y = 0$

This is a Cauchy-Euler equation which is solved by the substitution $y = t^r$.

Differentiating we have

$y' = rt^{r-1}$

$y'' = r(r-1)t^{r-2}$

Then

$2t^2r(r-1)t^{r-2} + 3trt^{r-1} - t^r = 0$

$t^r(2r^2 - 2r + 3r - 1) = 0$

$2r^2 + r - 1 = 0$

Use the quadratic formula

$$r = \frac{-1 \pm \sqrt{1 - 4(2)(-1)}}{2(2)} = \frac{-1 \pm 3}{4}$$

We have two distinct real roots

$$r_1 = \frac{-1+3}{4} = \frac{1}{2}$$

$$r_2 = \frac{-1-3}{4} = -1$$

The fundamental set of solutions of the equation

$$y = c_1(\sqrt{t}) + c_2\left(\frac{1}{t}\right), t > 0$$

Yes, the functions $y_1(t) = \sqrt{t}$ and $y_2(t) = 1/t$ form a fundamental set of solutions of the equation $2t^2y'' + 3ty' - y = 0$, on the interval $t \in (,\infty)$.