Answer on Question #74052 – Math – Calculus

Question

Trace the curve $x(y^2+4)=8$, stating all the points used for doing so.

Solution

1. Symmetry
$x(y^2+4)=8$
All powers of y in the equation are even.

$$x((-y)^2+4)=x(y^2+4)$$

Therefore, curve is symmetrical about x- axis.
Curve is not symmetrical about y- axis.
Curve is not symmetrical about the line $y=x$.
Curve is not symmetrical about the line $y=-x$.
Curve is not symmetrical in opposite quadrants.

2. Origin
$x(y^2+4)=8$
$(0,0): 0((0)^2+4)=8$
$4=8,$ False
A curve does not pass through the origin.

3. Intercepts
$x(y^2+4)=8$
Intersection with x- axis: Put $y=0$ in the equation of the curve and solve the resulting equation
$x(0+4)=8 \Rightarrow x=2$
Intersection with x- axis: $Point\ (2,0)$

Intersection with y- axis: Put $x=0$ in the equation of the curve and solve the resulting equation
$0(y^2+4)=8 \Rightarrow No\ solutions.$
There is no intersection with y- axis.

Tangent to the curve
In order to find the tangent(s) at the point $(2,0)$, we should shift the origin to $(2,0)$ and then the tangent(s) at this new origin will be obtained by equating to zero the lowest degree term.
Tangent at the point $(2,0)$:

$$x=2$$

4. Asymptotes
$x(y^2 + 4) = 8$
Asymptote(s) parallel to y- axis
Equal to zero the coefficient of the highest degree terms in y

$$x = 0 \quad \text{Vertical asymptote}$$

Asymptote(s) parallel to x- axis
Equal to zero the coefficient of the highest degree terms in x

$$y^2 + 4 = 0 \quad \text{No solution}$$

There is no horizontal asymptote.

Oblique asymptote(s)
There is no oblique asymptote.

5. Regions where no Part of the curve lies
$x(y^2 + 4) = 8$
Since $y^2 + 4 > 0$, we have $x > 0$
$y^2 + 4 = \frac{8}{x}$
Since $y^2 + 4 \geq 4, x > 0,$ we $x \leq 2,$
Therefore, $0 < x \leq 2, y \in R$.
When $x \to 0^+, y \to \pm\infty$

6. First derivative
$x(y^2 + 4) = 8$
Differentiate both sides with respect to x and use the Chain rule

$$y^2 + 4 + x(2y)\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{y^2 + 4}{2xy}$$

$$\frac{dy}{dx} = -\frac{\frac{8}{x}}{2xy}$$

$$\frac{dy}{dx} = -\frac{4}{x^2 y}$$

$If\ y > 0, \frac{dy}{dx} < 0, y\ decreases$
$If\ y < 0, \frac{dy}{dx} > 0, y\ increases$
$If\ x \to 2^-, \frac{dy}{dx} \to \pm\infty$

The tangent at the point $(2,0)$ is the straight line parallel to the y −axis.

7. Second derivative

$$x(y^2+4)=8$$

$$\frac{dy}{dx}=-\frac{4}{x^2y}$$

Differentiate both sides with respect to x and use the Chain rule

$$\frac{d^2y}{dx^2}=4\frac{2xy+x^2\frac{dy}{dx}}{x^4y^2}$$

$$\frac{d^2y}{dx^2}=4\frac{2xy+x^2\left(-\frac{4}{x^2y}\right)}{x^4y^2}$$

$$\frac{d^2y}{dx^2}=4\frac{2xy-\frac{4}{y}}{x^4y^2}$$

$$\frac{d^2y}{dx^2}=8\frac{xy^2-2}{x^4y^3}$$

$$\frac{d^2y}{dx^2}=0 \Rightarrow 8\frac{xy^2-2}{x^4y^3}=0$$

$$xy^2-2=0$$

$$x\left(\frac{8}{x}-4\right)-2=0$$

$$8-4x-2=0$$

$$x=\frac{3}{2}$$

$$\frac{3}{2}(y^2+4)=8$$

$$y^2+4=\frac{16}{3}$$

$$y^2=\frac{4}{3}$$

$$y=\pm\frac{2\sqrt{3}}{3}$$

$$If\ y<-\frac{2\sqrt{3}}{3}, 0<x<\frac{3}{2}, \frac{d^2y}{dx^2}<0, the\ graph\ concaves\ down$$

$$If\ -\frac{2\sqrt{3}}{3}<y<0, \frac{3}{2}<x<2, \frac{d^2y}{dx^2}>0, the\ graph\ concaves\ up$$

$$If\ 0<y<\frac{2\sqrt{3}}{3}, \frac{3}{2}<x<2, \frac{d^2y}{dx^2}<0, the\ graph\ concaves\ down$$

$$If\ y > \frac{2\sqrt{3}}{3}\ , 0 < x < \frac{3}{2}\ , \frac{d^2y}{dx^2} > 0, the\ graph\ concaves\ up$$