Answer on Question #73784 – Math – Statistics and Probability

Question

Consider the following five data points:

x	-1.0	0.0	1.0	2.0	3.0
y	-1.0	1.0	1.0	2.5	3.5

a. Use regression analysis to calculate by hand the estimated coefficients of the equation $y = b + ax$.

b. Compute the coefficient of determination.

c. What is the predicted value of y for $x = 1.0$? For $x = 3.5$?

Solution

a. Regression equation of y on x:

$$y = \mu_y + \frac{Cov(x,y)}{{\sigma_x}^2}(x - \mu_x).$$

where μ_x and μ_y are mean values of x and y, ${\sigma_x}^2$ is a variance of x and $Cov(x,y)$ is a covariance between x and y .

Let us calculate the necessary values:

$$\mu_x = E[X] = \frac{-1.0 + 0.0 + 1.0 + 2.0 + 3.0}{5} = 1.0$$

$$\mu_y = E[Y] = \frac{-1.0 + 1.0 + 1.0 + 2.5 + 3.5}{5} = 1.4$$

$$Cov(x,y) = E[XY] - E[X]E[Y]$$
$$= \frac{(-1.0)\cdot(-1.0) + 0.0\cdot 1.0 + 1.0\cdot 1.0 + 2.0\cdot 2.5 + 3.0\cdot 3.5}{5} - 1.0\cdot 1.4 = 2.1$$

$${\sigma_x}^2 = Var[X] = \frac{(-1.0-1.0)^2 + (0.0-1.0)^2 + (1.0-1.0)^2 + (2.0-1.0)^2 + (3.0-1.0)^2}{5}$$
$$= 2.0$$

Therefore, the regression equation is

$$y = 1.4 + \frac{2.1}{2.0}(x - 1.0)$$

or

$$y = 1.05x + 0.35$$

b. Let us first calculate the predicted values $\hat{y}_i$ of dependent variable by formula

$$\hat{y}_i = 1.05x_i + 0.35$$

The results we put in the Table 1:

x	-1.0	0.0	1.0	2.0	3.0
y	-1.0	1.0	1.0	2.5	3.5
$\hat{y}$	-0.7	0.35	1.4	2.45	3.5

The coefficient of determination R^2 is defined as

$$R^2 = 1 - \frac{SS_{res}}{SS_{tot}},$$

where

$$SS_{res} = \sum_i (y_i - \hat{y}_i)^2$$
$$= (-1.0 + 0.7)^2 + (1.0 - 0.35)^2 + (1.0 - 1.4)^2 + (2.5 - 2.45)^2 + (3.5 - 3.5)^2$$
$$= 0.675;$$

$$SS_{tot} = \sum_i (y_i - \mu_y)^2$$
$$= (-1.0 - 1.4)^2 + (1.0 - 1.4)^2 + (1.0 - 1.4)^2 + (2.5 - 1.4)^2 + (3.5 - 1.4)^2$$
$$= 11.7$$

Therefore,

$$R^2 = 1 - \frac{0.675}{11.7} = 0.94$$

c. The predicted values of y is calculated in Table 1. So the predicted value of y for $x = 1.0$ is equal to $1.05 \cdot 1.0 + 0.35 = 1.4$ and the predicted value of y for $x = 3.5$ is equal to $1.05 \cdot 3.5 + 0.35 = 4.025$

Answer: a. $y = 1.05x + 0.35, b = 0.35, a = 1.05$; **b.** $R^2 = 0.94$; **c.** 1.4 and 4.025.