Answer on Question #73543 – Math – Calculus

Question

Evaluate the integral by converting to polar coordinate integrate 0 to √3 double integrate y to√(4-y^2) dx.dy/(4+ x^2+y^2)

Solution

Given $\int_0^{\sqrt{3}} \int_y^{\sqrt{4-y^2}} \frac{dxdy}{4+x^2+y^2}$.

By the definition of the polar coordinates $x = r\cos\theta, y = r\sin\theta,\ x^2 + y^2 = r^2,\ dxdy = rdrd\theta$

First of all limits of y are given 0 and $\sqrt{3}$ here $y = r\sin\theta,$ when y = 0 then $\theta = 0$, y = $\sqrt{3}$ then $\theta = \frac{\pi}{3}$

Similarly here

$x = rcos\theta, x = \sqrt{4-y^2}, x^2 + y^2 = r^2, r^2 = 4, r = 2.$

For the region of integration, r varies from 0 to 2 and $\theta = 0$ *and* $\theta = \frac{\pi}{3}$

$$\int_0^{\frac{\pi}{3}} \int_0^2 \frac{rdrd\theta}{4+r^2}$$

$$= \frac{1}{2}\int_0^{\frac{\pi}{3}} \int_0^2 \frac{2rdrd\theta}{4+r^2} = \frac{1}{2}\int_0^{\frac{\pi}{3}} \log\left(4+r^2\right)\Big|_0^2 d\theta$$

$$= \frac{1}{2}\int_0^{\frac{\pi}{3}} (\log 8 - \log 4) d\theta = \frac{1}{2}(\log 8 - \log 4)\int_0^{\frac{\pi}{3}} d\theta$$

$$= \frac{1}{2}(\log 2)\int_0^{\frac{\pi}{3}} d\theta$$

$$\frac{\log 2}{2}\frac{\pi}{3} = \log 2\left(\frac{\pi}{6}\right)$$

Answer: $\frac{\pi}{6}\log(2)$.