Answer on Question #73487 – Math – Calculus

Question

Check whether there exists a continuously differentiable function g defined by f(x,y) =0 in the neighborhood of x=3 such that g(3) =1/3 .

Solution

First of all, check the given function is differentiable function or not. Suppose that $f:(a,b) \rightarrow R \;\; and \;\; a < c < b$, $y = g(x)$, $f(x,y) = g(x) - y$

Then there exists a differentiable function g at c with derivative $g^{'}(x)$ if

$$\lim_{h \to 0} \frac{g(c+h)-g(c)}{h} = g'(c).$$

The domain of f ′ is the set of points c ∈ (a, b) for which this limit exists. If the limit exists for every c ∈ (a, b) then we say that f is differentiable on (a, b).

Let $g(x) = \frac{1}{x}, \;\; x \neq 0 \;\; g^{'}(x) = -\frac{1}{x^2}$ since

$$\lim_{h \to 0} \left[\frac{\frac{1}{(c+h)} - \frac{1}{(c)}}{h} \right] = \lim_{h \to 0} \left[\frac{c-c-h}{hc(c+h)} \right] = \lim_{h \to 0} \left[\frac{-h}{hc(c+h)} \right]$$

$$\lim_{h \to 0} \left[\frac{-1}{c(c+h)} \right] = \frac{-1}{c^2}$$

Hence the function $g(x) = \frac{1}{x}$ is differentiable function in the neighborhood of $x = 3$.. Therefore, it is true there exists a continuously differentiable function g defined by f(x,y) =0 in the neighborhood of x=3 such that g(3) =1/3 .