Answer on Question #73398 – Math – Algebra

Question

Polynomials $p(x) = 4x^3 - 2x^2 + px + 5$ and $p(x) = x^3 + 6x^2 + p$, leave the remainders a and b respectively, when divided by $(x - 2)$. Find the value of p, if $a + b = 0$.

Solution

The Remainder Theorem
Let $P(x)$ be any polynomial of degree greater than or equal one and let c be any real number. If $P(x)$ is divided by the linear polynomial $(x - c)$, then the remainder is $P(c)$.

If $p(x)$ is divided by the linear polynomial $(x - 2)$, then the remainder is $p(2)$
$remainder = p(2) = 4(2)^3 - 2(2)^2 + p(2) + 5 = 2p + 29$
We have that $2p + 29 = a$

If $q(x)$ is divided by the linear polynomial $(x - 2)$, then the remainder is $q(2)$
$remainder = q(2) = (2)^3 + 6(2)^2 + p = p + 32$
We have that $p + 32 = b$

If $a + b = 0$, then
$2p + 29 + p + 32 = 0$
$3p = -61$
$p = -\frac{61}{3}$
Check

$$p(x) = 4x^3 - 2x^2 - \frac{61}{3}x + 5$$

$$\frac{p(x)}{x-2} = \frac{4x^3 - 2x^2 - \frac{61}{3}x + 5}{x-2} = 4x^2 + 6x - \frac{25}{3} + \frac{-\frac{35}{3}}{x-2}$$

$$q(x) = x^3 + 6x^2 - \frac{61}{3}$$

$$\frac{q(x)}{x-2} = \frac{x^3 + 6x^2 - \frac{61}{3}}{x-2} = x^2 + 8x + 16 + \frac{\frac{35}{3}}{x-2}$$

$$-\frac{35}{3} + \frac{35}{3} = 0$$

Answer: $p = -\frac{61}{3}$.