Answer on Question #73255 – Math – Calculus

Question

Check whether the function f : R2 to R defined by f(x,y)=2x^4 -3x^2y +y^2 has an extrema at(0,0).

Solution

$$\frac{\partial f}{\partial x} = 2(4x^3) - 3y(2x) = 8x^3 - 6xy$$

$$\frac{\partial f}{\partial x}(0,0) = 0$$

$$\frac{\partial f}{\partial y} = -3x^2(1) + 2y = 2y - 3x^2$$

$$\frac{\partial f}{\partial y}(0,0) = 0$$

$$A = \frac{\partial}{\partial x}(8x^3 - 6xy) = 8(3x^2) - 6y = 24x^2 - 6y.$$

$$A(0,0) = 0$$

$$B = \frac{\partial}{\partial y}(8x^3 - 6xy) = -6x.$$

$$B(0,0) = 0$$

$$C = \frac{\partial}{\partial y}(2y - 3x^2) = 2.$$

$$C(0,0) = 2$$

Thus,

$$B^2 - AC = 0^2 - 0(2) = 0.$$

Thus, the type of critical point cannot be determined from the values of the partial derivatives.

But, returning to f, we can see that (0,0) is a minimum for f if we restrict our attention on x-axis, and (0,0) is a minimum for f if we restrict our attention on y-axis.

Therefore, f(x,y) has a minimum at (0,0).