Answer on Question #73120 - Math - Statistics and Probability

Question

If I toss 2 coins at the same time for 60 times, what will be the probability distribution of heads or tails occurring per toss?

Solution

It does not matter how many times we toss 2 coins. The probability distribution of heads or tails occurring per one toss will be the same.

When we toss two coins simultaneously, then the possible outcomes are: (two heads) or (one head and one tail) or (two tails) i.e., in short (H, H) or (H, T) or (T, T) respectively, where H is denoted for head and T is denoted for tail.

The sample space is given by S = {HH, HT, TH, TT} .

Therefore,

$$p_1 = P(HH) = \frac{n(HH)}{n(S)} = \frac{1}{4};\ p_2 = P(\mathrm{HT} \cup \mathrm{TH}) = \frac{n(\mathrm{HT} \cup \mathrm{TH})}{n(S)} = \frac{2}{4} = \frac{1}{2};$$

$$p_3 = P(TT) = \frac{n(TT)}{n(S)} = \frac{1}{4}$$

Let X give the number of heads and Y the number of tails occurred by tossing of 2 coins simultaneously. The random variable X, as well as Y, has the following probability distribution

$$P(X = 0) = P(TT) = \frac{1}{4};\ P(X = 1) = P(\mathrm{HT} \cup \mathrm{TH}) = \frac{1}{2};\ P(X = 2) = P(HH) = \frac{1}{4};$$

$$P(Y = 0) = \frac{1}{4};\ P(Y = 1) = \frac{1}{2};\ P(Y = 2) = \frac{1}{4}.$$

Actually, X and Y follow the binomial distribution with n = 2 and $p = \frac{1}{2}$.

Let us now consider an experiment of tossing 2 coins for 60 times. Let E_1 denote the event that head occurs 0 times per one toss, E_2 denote the event that head occurs 1 time per one toss and E_3 the event that head occurs 2 times per one toss. Then we can consider how many times events E_1, E_2 and E_3 occur during these 60 times. It is the multinomial distribution. Let us denote the variable which is the number of occurred event E_i $(i = 1,2,3)$ as Z_i and p_i be a probability of E_i. The probability distribution of Z_1, Z_2, Z_3 is

$P(Z_1 = k_1, Z_2 = k_2, Z_3 = k_3) = \frac{60!}{k_1!k_2!k_3!} {p_1}^{k_1}{p_2}^{k_2}{p_3}^{k_3}$, where $k_i \in \{0,1,\cdots,60\}, i = 1,2,3$, and

$k_1 + k_2 + k_3 = 60.$