

Answer on Question #72982 – Math – Statistics and Probability

Question

Every secretary in a large company has been given the same assignment. The time to completion is normally distributed with mean time 90 minutes and standard deviation 12 minutes. Determine

- (a) the probability a secretary will take more than 110 minutes;
- (b) the probability a secretary will take between 85 to 90 minutes to complete assignment.

Solution

Let X be a random variable representing the time to completion of the assignment. X has a normal distribution with mean time $\mu=90$ minutes and standard deviation $\sigma=12$ minutes. Then the random variable $\frac{X-90}{12}$ has the standard normal distribution, that is $\frac{X-90}{12} \sim N(0, 1)$. Therefore,

- (a) the probability a secretary will take more than 110 minutes to complete the assignment is equal to

$$\Pr(X > 110) = \Pr\left(\frac{X - 90}{12} > \frac{110 - 90}{12}\right) = \Pr\left(\frac{X - 90}{12} > \frac{5}{3}\right) = 1 - \Phi(1.67) \approx 0.047,$$

where $\Phi(x)$ is a cumulative distribution function of the standard normal distribution;

- (b) the probability a secretary will take between 85 to 90 minutes to complete the assignment is equal to

$$\begin{aligned}\Pr(85 < X < 90) &= \Pr\left(\frac{85 - 90}{12} < \frac{X - 90}{12} < \frac{90 - 90}{12}\right) = \Pr\left(-\frac{5}{12} < \frac{X - 90}{12} < 0\right) = \\ &= \Phi(0) - \Phi(-0.42) \approx 0.163.\end{aligned}$$

Answer: (a) ≈ 0.047 ; (b) ≈ 0.163 .