Answer on Question #72902 – Math – Calculus

Question

The price P per unit at which a company can sell all that it produces is given by the function P(x) = 300 – 4x. The cost function is c(x) = 500 + 28x where x is the number of units produced. Find x so that the profit is maximum.

Solution

Profit function = Revenue function – Cost function

$$P(x) = R(x) - C(x)$$

$R(x)$ – Revenue function; $C(x)$ – Cost function.

The revenue function can be found as (price per unit) multiplied by (quantity of units). Therefore

$$R(x) = P(x) \cdot x$$

Therefore $R(x) = p(x) \cdot x = (300 - 4x)x = 300x - 4x^2$. As result get profit function

$$P(x) = 300x - 4x^2 - 500 - 28x = 272x - 4x^2 - 500$$

To find the maximum profit, we find the derivative $P'(x)$ and equate it to zero.

$$P'(x) = 272 - 8x = 0$$

$$272 - 8x = 0 \to x = 34$$

The value of the derivative for $x < 34$ is $P'(x) > 0$; the value of the derivative for $x > 34$ is $P'(x) < 0$. Hence, for x = 34, the profit takes the maximum value.

$$P(34) = 272 \cdot 34 - 4 \cdot 34^2 - 500 = 4124$$

Answer: The profit takes the maximum value 4124 for $x = 34$.