Answer on Question #72883 – Math – Calculus

Question

Solve

$$\int_1^2 \sqrt{1+\frac{1}{t^2}+(\ln t)^2+2\ln t+1}\,dt$$

Solution

$$\int_1^2 \sqrt{1+\frac{1}{t^2}+(\ln t)^2+2\ln t+1}\,dt$$

The approximation was obtained through numerical integration.
The absolute error of an approximation
Left Riemann Sum or Right Riemann Sum

$$|E_n| \le \frac{b-a}{n}\left(f(b)-f(a)\right)$$

Midpoint Rule

$$|E_n| \le \frac{(b-a)^3}{24n^2}\left\{\max_{a\le x\le b}|f''(x)|\right\}$$

Trapezoid Rule

$$|E_n| \le \frac{(b-a)^3}{12n^2}\left\{\max_{a\le x\le b}|f''(x)|\right\}$$

Simpson's Rule

$$|E_n| \le \frac{(b-a)^5}{180n^4}\left\{\max_{a\le x\le b}\left|f^{(4)}(x)\right|\right\}$$

Left Riemann Sum

$$\int_a^b f(t)\,dt \approx \Delta t\left(f(t_0)+f(t_1)+f(t_2)+\cdots+f(t_{n-1})\right)$$

$$\Delta t = \frac{b-a}{n}$$

We have that $a=1, b=2, n=10$.
Therefore,

$$\Delta t = \frac{2-1}{10} = \frac{1}{10}$$

Divide interval $[1,2]$ into $n=10$ subintervals of length $\Delta t = 0.1$ with the following endpoints

$a = 1, 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9, 2 = b$

$$f(t_0) = f(1) = \sqrt{1+\frac{1}{1^2}+(\ln 1)^2+2\ln 1+1} = \sqrt{3} \approx 1.732051$$

$$f(t_1) = f(1.1) = \sqrt{1+\frac{1}{1.1^2}+(\ln 1.1)^2+2\ln 1.1+1} \approx 1.739583$$

$$f(t_2) = f(1.2) = \sqrt{1 + \frac{1}{1.2^2} + (\ln 1.2)^2 + 2\ln 1.2 + 1} \approx 1.758502$$

$$f(t_3) = f(1.3) = \sqrt{1 + \frac{1}{1.3^2} + (\ln 1.3)^2 + 2\ln 1.3 + 1} \approx 1.784735$$

$$f(t_4) = f(1.4) = \sqrt{1 + \frac{1}{1.4^2} + (\ln 1.4)^2 + 2\ln 1.4 + 1} \approx 1.815589$$

$$f(t_5) = f(1.5) = \sqrt{1 + \frac{1}{1.5^2} + (\ln 1.5)^2 + 2\ln 1.5 + 1} \approx 1.849264$$

$$f(t_6) = f(1.6) = \sqrt{1 + \frac{1}{1.6^2} + (\ln 1.6)^2 + 2\ln 1.6 + 1} \approx 1.884552$$

$$f(t_7) = f(1.7) = \sqrt{1 + \frac{1}{1.7^2} + (\ln 1.7)^2 + 2\ln 1.7 + 1} \approx 1.920636$$

$$f(t_8) = f(1.8) = \sqrt{1 + \frac{1}{1.8^2} + (\ln 1.8)^2 + 2\ln 1.8 + 1} \approx 1.956964$$

$$f(t_9) = f(1.9) = \sqrt{1 + \frac{1}{1.9^2} + (\ln 1.9)^2 + 2\ln 1.9 + 1} \approx 1.993161$$

$$\int_1^2 \sqrt{1 + \frac{1}{t^2} + (\ln t)^2 + 2\ln t + 1}\, dt \approx 0.1(1.732051 + 1.739583 +$$
$$+1.758502 + 1.784735 + 1.815589 + 1.849264 + 1.884552 +$$
$$+1.920636 + 1.956964 + 1.993161) \approx 1.8435$$

Right Riemann Sum

$$\int_a^b f(t)\, dt \approx \Delta t\big(f(t_1) + f(t_2) + f(t_3) + \cdots + f(t_n)\big)$$

$$\Delta t = \frac{b-a}{n}$$

We have that $a = 1, b = 2, n = 10$.

Therefore,

$$\Delta t = \frac{2-1}{10} = \frac{1}{10}$$

Divide interval $[1, 2]$ into $n = 10$ subintervals of length $\Delta t = 0.1$ with the following endpoints

$$a = 1, 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9, 2 = b$$

$$f(t_1) = f(1.1) = \sqrt{1 + \frac{1}{1.1^2} + (\ln 1.1)^2 + 2\ln 1.1 + 1} \approx 1.739583$$

$$f(t_2) = f(1.2) = \sqrt{1 + \frac{1}{1.2^2} + (\ln 1.2)^2 + 2\ln 1.2 + 1} \approx 1.758502$$

$$f(t_3) = f(1.3) = \sqrt{1 + \frac{1}{1.3^2} + (\ln 1.3)^2 + 2\ln 1.3 + 1} \approx 1.784735$$

$$f(t_4) = f(1.4) = \sqrt{1 + \frac{1}{1.4^2} + (\ln 1.4)^2 + 2\ln 1.4 + 1} \approx 1.815589$$

$$f(t_5) = f(1.5) = \sqrt{1 + \frac{1}{1.5^2} + (\ln 1.5)^2 + 2\ln 1.5 + 1} \approx 1.849264$$

$$f(t_6) = f(1.6) = \sqrt{1 + \frac{1}{1.6^2} + (\ln 1.6)^2 + 2\ln 1.6 + 1} \approx 1.884552$$

$$f(t_7) = f(1.7) = \sqrt{1 + \frac{1}{1.7^2} + (\ln 1.7)^2 + 2\ln 1.7 + 1} \approx 1.920636$$

$$f(t_8) = f(1.8) = \sqrt{1 + \frac{1}{1.8^2} + (\ln 1.8)^2 + 2\ln 1.8 + 1} \approx 1.956964$$

$$f(t_9) = f(1.9) = \sqrt{1 + \frac{1}{1.9^2} + (\ln 1.9)^2 + 2\ln 1.9 + 1} \approx 1.993161$$

$$f(t_{10}) = f(2) = \sqrt{1 + \frac{1}{2^2} + (\ln 2)^2 + 2\ln 2 + 1} \approx 2.028977$$

$$\int_1^2 \sqrt{1 + \frac{1}{t^2} + (\ln t)^2 + 2\ln t + 1}\, dt \approx 0.1(1.739583 + 1.758502 +$$
$$+1.784735 + 1.815589 + 1.849264 + 1.884552 + 1.920636 +$$
$$+1.956964 + 1.993161 + 2.028977) \approx 1.8732$$

Trapezoid Rule

$$\int_a^b f(t)\,dt \approx$$

$$\approx \frac{\Delta t}{2}\left(f(t_0) + 2f(t_1) + 2f(t_2) + \cdots + 2f(t_{n-1}) + f(t_n)\right)$$

$$\Delta t = \frac{b-a}{n}$$

We have that $a = 1, b = 2, n = 20.$

Therefore,

$$\Delta t = \frac{2-1}{10} = \frac{1}{10}$$

Divide interval $[1, 2]$ into $n = 10$ subintervals of length $\Delta t = 0.1$ with the following endpoints

$a = 1, 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9, 2 = b$

$$f(t_0) = f(1) = \sqrt{1 + \frac{1}{1^2} + (\ln 1)^2 + 2\ln 1 + 1} \approx 1.732051$$

$$2f(t_1) = 2f(1.1) = 2\sqrt{1 + \frac{1}{1.1^2} + (\ln 1.1)^2 + 2\ln 1.1 + 1} \approx 3.479167$$

$$2f(t_2) = 2f(1.2) = 2\sqrt{1 + \frac{1}{1.2^2} + (\ln 1.2)^2 + 2\ln 1.2 + 1} \approx 3.517004$$

$$2f(t_3) = 2f(1.3) = 2\sqrt{1 + \frac{1}{1.3^2} + (\ln 1.3)^2 + 2\ln 1.3 + 1} \approx 3.569470$$

$$2f(t_4) = 2f(1.4) = 2\sqrt{1 + \frac{1}{1.4^2} + (\ln 1.4)^2 + 2\ln 1.4 + 1} \approx 3.631177$$

$$2f(t_5) = 2f(1.5) = 2\sqrt{1 + \frac{1}{1.5^2} + (\ln 1.5)^2 + 2\ln 1.5 + 1} \approx 3.698528$$

$$2f(t_6) = 2f(1.6) = 2\sqrt{1 + \frac{1}{1.6^2} + (\ln 1.6)^2 + 2\ln 1.6 + 1} \approx 3.769104$$

$$2f(t_7) = 2f(1.7) = 2\sqrt{1 + \frac{1}{1.7^2} + (\ln 1.7)^2 + 2\ln 1.7 + 1} \approx 3.841272$$

$$2f(t_8) = 2f(1.8) = 2\sqrt{1 + \frac{1}{1.8^2} + (\ln 1.8)^2 + 2\ln 1.8 + 1} \approx 3.913928$$

$$2f(t_9) = 2f(1.9) = 2\sqrt{1 + \frac{1}{1.9^2} + (\ln 1.9)^2 + 2\ln 1.9 + 1} \approx 3.986323$$

$$f(t_{10}) = f(2) = \sqrt{1 + \frac{1}{2^2} + (\ln 2)^2 + 2\ln 2 + 1} \approx 2.028977$$

$$\int_1^2 \sqrt{1 + \frac{1}{t^2} + (\ln t)^2 + 2\ln t + 1}\, dt \approx \frac{0.1}{2}(1.732051 + 3.479167 +$$
$$+3.517004 + 3.569470 + 3.631177 + 3.698528 + 3.769104 +$$
$$+3.841272 + 3.913928 + 3.986323 + 2.028977) \approx 1.85835$$

Midpoint Rule

$$\int_a^b f(t)\, dt \approx$$
$$\approx \Delta t \left(f\left(\frac{t_0 + t_1}{2}\right) + f\left(\frac{t_1 + t_2}{2}\right) + \cdots + f\left(\frac{t_{n-2} + t_{n-1}}{2}\right) + f\left(\frac{t_{n-1} + t_n}{2}\right) \right)$$
$$\Delta t = \frac{b - a}{n}$$

We have that $a = 1, b = 2, n = 10$.

Therefore,

$$\Delta t = \frac{2 - 1}{10} = 0.1$$

Divide interval $[1, 2]$ into $n = 10$ subintervals of length $\Delta t = 0.1$ with the following endpoints

$a = 1, 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9, 2 = b$

$$f\left(\frac{t_0 + t_1}{2}\right) = f\left(\frac{1 + 1.1}{2}\right) = f(1.05) =$$
$$= \sqrt{1 + \frac{1}{1.05^2} + (\ln 1.05)^2 + 2\ln 1.05 + 1} \approx 1.734068$$

$$f\left(\frac{t_1 + t_2}{2}\right) = f\left(\frac{1.1 + 1.2}{2}\right) = f(1.15) =$$
$$= \sqrt{1 + \frac{1}{1.15^2} + (\ln 1.15)^2 + 2\ln 1.15 + 1} \approx 1.747913$$

$$f\left(\frac{t_2 + t_3}{2}\right) = f\left(\frac{1.2 + 1.3}{2}\right) = f(1.25) =$$
$$= \sqrt{1 + \frac{1}{1.25^2} + (\ln 1.25)^2 + 2\ln 1.25 + 1} \approx 1.770898$$

$$f\left(\frac{t_3+t_4}{2}\right)=f\left(\frac{1.3+1.4}{2}\right)=f(1.35)=$$
$$=\sqrt{1+\frac{1}{1.35^2}+(\ln 1.35)^2+2\ln 1.35+1}\approx 1.799714$$
$$f\left(\frac{t_4+t_5}{2}\right)=f\left(\frac{1.4+1.5}{2}\right)=f(1.45)=$$
$$=\sqrt{1+\frac{1}{1.45^2}+(\ln 1.45)^2+2\ln 1.45+1}\approx 1.832160$$
$$f\left(\frac{t_5+t_6}{2}\right)=f\left(\frac{1.5+1.6}{2}\right)=f(1.55)=$$
$$=\sqrt{1+\frac{1}{1.55^2}+(\ln 1.55)^2+2\ln 1.55+1}\approx 1.866765$$
$$f\left(\frac{t_6+t_7}{2}\right)=f\left(\frac{1.6+1.7}{2}\right)=f(1.65)=$$
$$=\sqrt{1+\frac{1}{1.65^2}+(\ln 1.65)^2+2\ln 1.65+1}\approx 1.902534$$
$$f\left(\frac{t_7+t_8}{2}\right)=f\left(\frac{1.7+1.8}{2}\right)=f(1.75)=$$
$$=\sqrt{1+\frac{1}{1.75^2}+(\ln 1.75)^2+2\ln 1.75+1}\approx 1.938797$$

$$f\left(\frac{t_8+t_9}{2}\right)=f\left(\frac{1.8+1.9}{2}\right)=f(1.85)=$$
$$=\sqrt{1+\frac{1}{1.85^2}+(\ln 1.85)^2+2\ln 1.85+1}\approx 1.975097$$
$$f\left(\frac{t_9+t_{10}}{2}\right)=f\left(\frac{1.9+2}{2}\right)=f(1.95)=$$
$$=\sqrt{1+\frac{1}{1.95^2}+(\ln 1.95)^2+2\ln 1.95+1}\approx 2.011123$$

$$\int_1^2\sqrt{1+\frac{1}{t^2}+(\ln t)^2+2\ln t+1}\,dt\approx$$
$$\approx 0.1(1.734068+1.747913+1.770898+1.799714+1.832160+$$
$$+1.866765+1.902534+1.938797+1.975097+2.011123)\approx 1.85791$$

Simpson's Rule

$$\int_a^b f(t)\,dt \approx$$

$$\approx \frac{\Delta t}{3}\left(f(t_0) + 4f(t_1) + 2f(t_2) + 4f(t_3) + 2f(t_4) + \cdots + \right.$$

$$\left. +4f(t_{n-2}) + 2f(t_{n-1}) + f(t_n)\right)$$

$$\Delta t = \frac{b-a}{n}$$

We have that $a = 1, b = 2, n = 10$.

Therefore,

$$\Delta t = \frac{2-1}{10} = 0.1$$

Divide interval $[1, 2]$ into $n = 10$ subintervals of length $\Delta t = 0.1$ with the following endpoints

$a = 1, 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9, 2 = b$

$$f(t_0) = f(1) = \sqrt{1 + \frac{1}{1^2} + (\ln 1)^2 + 2\ln 1 + 1} \approx 1.732051$$

$$4f(t_1) = 4f(1.1) = 4\sqrt{1 + \frac{1}{1.1^2} + (\ln 1.1)^2 + 2\ln 1.1 + 1} \approx 6.958334$$

$$2f(t_2) = 2f(1.2) = 2\sqrt{1 + \frac{1}{1.2^2} + (\ln 1.2)^2 + 2\ln 1.2 + 1} \approx 3.517004$$

$$4f(t_3) = 4f(1.3) = 4\sqrt{1 + \frac{1}{1.3^2} + (\ln 1.3)^2 + 2\ln 1.3 + 1} \approx 7.138941$$

$$2f(t_4) = 2f(1.4) = 2\sqrt{1 + \frac{1}{1.4^2} + (\ln 1.4)^2 + 2\ln 1.4 + 1} \approx 3.631177$$

$$4f(t_5) = 4f(1.5) = 4\sqrt{1 + \frac{1}{1.5^2} + (\ln 1.5)^2 + 2\ln 1.5 + 1} \approx 7.397055$$

$$2f(t_6) = 2f(1.6) = 2\sqrt{1 + \frac{1}{1.6^2} + (\ln 1.6)^2 + 2\ln 1.6 + 1} \approx 3.769104$$

$$4f(t_7) = 4f(1.7) = 4\sqrt{1 + \frac{1}{1.7^2} + (\ln 1.7)^2 + 2\ln 1.7 + 1} \approx 7.682545$$

$$2f(t_8) = 2f(1.8) = 2\sqrt{1 + \frac{1}{1.8^2} + (\ln 1.8)^2 + 2\ln 1.8 + 1} \approx 3.913928$$

$$4f(t_9) = 4f(1.9) = 4\sqrt{1 + \frac{1}{1.9^2} + (\ln 1.9)^2 + 2\ln 1.9 + 1} \approx 7.972646$$

$$f(t_{10}) = f(2) = \sqrt{1 + \frac{1}{2^2} + (\ln 2)^2 + 2\ln 2 + 1} \approx 2.028977$$

$$\int_1^2 \sqrt{1 + \frac{1}{t^2} + (\ln t)^2 + 2\ln t + 1}\, dt \approx \frac{0.1}{3}(1.732051 + 6.958334 +$$
$$+3.517004 + 7.138941 + 3.631177 + 7.397055 + 3.769104 +$$
$$+7.682545 + 3.913928 + 7.972646 + 2.028977) \approx 1.85806$$

We have that

$$\int_1^2 \sqrt{1 + \frac{1}{t^2} + (\ln t)^2 + 2\ln t + 1}\, dt \approx 1.858$$