Answer on Question #72782 – Math – Calculus
Question

Solve

$$\int_1^2 \sqrt{1 + \frac{1}{t^2} + (\ln t)^2 + 2\ln t + 1} \, dt$$

Solution

$$\int_1^2 \sqrt{1 + \frac{1}{t^2} + (\ln t)^2 + 2\ln t + 1} \, dt$$

The approximation was obtained through numerical integration.
Left Riemann Sum (n=20):1.85070557000951
Right Riemann Sum (n=20):1.86555187603946
Midpoint Rule (n=20):1.85801770720187
Trapezoid Rule (n=20):1.85812872302449
Simpson's Rule (n=20):1.85805495103159

$$\int_1^2 \sqrt{1 + \frac{1}{t^2} + (\ln t)^2 + 2\ln t + 1} \, dt \approx 1.85805$$

The absolute error of an approximation
Left Riemann Sum or Right Riemann Sum

$$|E_n| \leq \frac{b-a}{n}(f(b) - f(a))$$

Midpoint Rule

$$|E_n| \leq \frac{(b-a)^3}{24n^2} \left\{ \max_{a \leq x \leq b} |f''(x)| \right\}$$

Trapezoid Rule

$$|E_n| \leq \frac{(b-a)^3}{12n^2} \left\{ \max_{a \leq x \leq b} |f''(x)| \right\}$$

Simpson's Rule

$$|E_n| \leq \frac{(b-a)^5}{180n^4} \left\{ \max_{a \leq x \leq b} |f^{(4)}(x)| \right\}$$

Trapezoid Rule

$$\int_a^b f(t) \, dt \approx$$

$$\approx \frac{\Delta t}{2} \left(f(t_0) + 2f(t_1) + 2f(t_2) + \cdots + 2f(t_{n-1}) + f(t_n) \right)$$

$$\Delta t = \frac{b-a}{n}$$

We have that $a = 1, b = 2, n = 10$.
Therefore,

$$\Delta t = \frac{2-1}{10} = 0.1$$

Divide interval $[1, 2]$ into $n = 10$ subintervals of length $\Delta t = 0.1$ with the following endpoints

$$a = 1, 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9, 2 = b$$

$$f(t_0) = f(1) = \sqrt{1 + \frac{1}{1^2} + (\ln 1)^2 + 2\ln 1 + 1} \approx 1.732051$$

$$2f(t_1) = 2f(1.1) = 2\sqrt{1 + \frac{1}{1.1^2} + (\ln 1.1)^2 + 2\ln 1.1 + 1} \approx 3.479167$$

$$2f(t_2) = 2f(1.2) = 2\sqrt{1 + \frac{1}{1.2^2} + (\ln 1.2)^2 + 2\ln 1.2 + 1} \approx 3.517004$$

$$2f(t_3) = 2f(1.3) = 2\sqrt{1 + \frac{1}{1.3^2} + (\ln 1.3)^2 + 2\ln 1.3 + 1} \approx 3.569470$$

$$2f(t_4) = 2f(1.4) = 2\sqrt{1 + \frac{1}{1.4^2} + (\ln 1.4)^2 + 2\ln 1.4 + 1} \approx 3.631177$$

$$2f(t_5) = 2f(1.5) = 2\sqrt{1 + \frac{1}{1.5^2} + (\ln 1.5)^2 + 2\ln 1.5 + 1} \approx 3.698528$$

$$2f(t_6) = 2f(1.6) = 2\sqrt{1 + \frac{1}{1.6^2} + (\ln 1.6)^2 + 2\ln 1.6 + 1} \approx 3.769104$$

$$2f(t_7) = 2f(1.7) = 2\sqrt{1 + \frac{1}{1.7^2} + (\ln 1.7)^2 + 2\ln 1.7 + 1} \approx 3.841272$$

$$2f(t_8) = 2f(1.8) = 2\sqrt{1 + \frac{1}{1.8^2} + (\ln 1.8)^2 + 2\ln 1.8 + 1} \approx 3.913928$$

$$2f(t_9) = 2f(1.9) = 2\sqrt{1 + \frac{1}{1.9^2} + (\ln 1.9)^2 + 2\ln 1.9 + 1} \approx 3.986323$$

$$f(t_{10}) = f(2) = \sqrt{1 + \frac{1}{2^2} + (\ln 2)^2 + 2\ln 2 + 1} \approx 2.028977$$

$$\int_1^2 \sqrt{1 + \frac{1}{t^2} + (\ln t)^2 + 2\ln t + 1} \, dt \approx \frac{0.1}{2}(1.732051 + 3.479167 +$$
$$+3.517004 + 3.569470 + 3.631177 + 3.698528 + 3.769104 +$$
$$+3.841272 + 3.913928 + 3.986323 + 2.028977) \approx 1.858$$

Midpoint Rule

$$\int_a^b f(t)\, dt \approx$$

$$\approx \Delta t \left(f\left(\frac{t_0 + t_1}{2}\right) + f\left(\frac{t_1 + t_2}{2}\right) + \cdots + f\left(\frac{t_{n-2} + t_{n-1}}{2}\right) + f\left(\frac{t_{n-1} + t_n}{2}\right) \right)$$

$$\Delta t = \frac{b - a}{n}$$

We have that $a = 1, b = 2, n = 10$.

Therefore,

$$\Delta t = \frac{2 - 1}{10} = 0.1$$

Divide interval $[1, 2]$ into $n = 10$ subintervals of length $\Delta t = 0.1$ with the following endpoints

$$a = 1, 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9, 2 = b$$

$$f\left(\frac{t_0 + t_1}{2}\right) = f\left(\frac{1 + 1.1}{2}\right) = f(1.05) =$$

$$= \sqrt{1 + \frac{1}{1.05^2} + (\ln 1.05)^2 + 2\ln 1.05 + 1} \approx 1.734068$$

$$f\left(\frac{t_1 + t_2}{2}\right) = f\left(\frac{1.1 + 1.2}{2}\right) = f(1.15) =$$

$$= \sqrt{1 + \frac{1}{1.15^2} + (\ln 1.15)^2 + 2\ln 1.15 + 1} \approx 1.747913$$

$$f\left(\frac{t_2 + t_3}{2}\right) = f\left(\frac{1.2 + 1.3}{2}\right) = f(1.25) =$$

$$= \sqrt{1 + \frac{1}{1.25^2} + (\ln 1.25)^2 + 2\ln 1.25 + 1} \approx 1.770898$$

$$f\left(\frac{t_3 + t_4}{2}\right) = f\left(\frac{1.3 + 1.4}{2}\right) = f(1.35) =$$

$$= \sqrt{1 + \frac{1}{1.35^2} + (\ln 1.35)^2 + 2\ln 1.35 + 1} \approx 1.799714$$

$$f\left(\frac{t_4 + t_5}{2}\right) = f\left(\frac{1.4 + 1.5}{2}\right) = f(1.45) =$$

$$= \sqrt{1 + \frac{1}{1.45^2} + (\ln 1.45)^2 + 2\ln 1.45 + 1} \approx 1.832160$$

$$f\left(\frac{t_5 + t_6}{2}\right) = f\left(\frac{1.5 + 1.6}{2}\right) = f(1.55) =$$

$$= \sqrt{1 + \frac{1}{1.55^2} + (\ln 1.55)^2 + 2\ln 1.55 + 1} \approx 1.866765$$

$$f\left(\frac{t_6 + t_7}{2}\right) = f\left(\frac{1.6 + 1.7}{2}\right) = f(1.65) =$$

$$= \sqrt{1 + \frac{1}{1.65^2} + (\ln 1.65)^2 + 2\ln 1.65 + 1} \approx 1.902534$$

$$f\left(\frac{t_7 + t_8}{2}\right) = f\left(\frac{1.7 + 1.8}{2}\right) = f(1.75) =$$

$$= \sqrt{1 + \frac{1}{1.75^2} + (\ln 1.75)^2 + 2\ln 1.75 + 1} \approx 1.938797$$

$$f\left(\frac{t_8 + t_9}{2}\right) = f\left(\frac{1.8 + 1.9}{2}\right) = f(1.85) =$$

$$= \sqrt{1 + \frac{1}{1.85^2} + (\ln 1.85)^2 + 2\ln 1.85 + 1} \approx 1.975097$$

$$f\left(\frac{t_9 + t_{10}}{2}\right) = f\left(\frac{1.9 + 2}{2}\right) = f(1.95) =$$

$$= \sqrt{1 + \frac{1}{1.95^2} + (\ln 1.95)^2 + 2\ln 1.95 + 1} \approx 2.011123$$

$$\int_1^2 \sqrt{1 + \frac{1}{t^2} + (\ln t)^2 + 2\ln t + 1}\, dt \approx$$
$$\approx 0.1(1.734068 + 1.747913 + 1.770898 + 1.799714 + 1.832160 +$$
$$+1.866765 + 1.902534 + 1.938797 + 1.975097 + 2.011123) \approx 1.858$$

Simpson's Rule
$$\int_a^b f(t)\, dt \approx$$
$$\approx \frac{\Delta t}{3}(f(t_0) + 4f(t_1) + 2f(t_2) + 4f(t_3) + 2f(t_4) + \cdots +$$
$$+4f(t_{n-2}) + 2f(t_{n-1}) + f(t_n))$$
$$\Delta t = \frac{b - a}{n}$$
We have that $a = 1, b = 2, n = 10$.
Therefore,
$$\Delta t = \frac{2 - 1}{10} = 0.1$$

Divide interval $[1, 2]$ into $n = 10$ subintervals of length $\Delta t = 0.1$ with the following endpoints

$a = 1, 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9, 2 = b$

$$f(t_0) = f(1) = \sqrt{1 + \frac{1}{1^2} + (\ln 1)^2 + 2 \ln 1 + 1} \approx 1.732051$$

$$4f(t_1) = 4f(1.1) = 4\sqrt{1 + \frac{1}{1.1^2} + (\ln 1.1)^2 + 2 \ln 1.1 + 1} \approx 6.958334$$

$$2f(t_2) = 2f(1.2) = 2\sqrt{1 + \frac{1}{1.2^2} + (\ln 1.2)^2 + 2 \ln 1.2 + 1} \approx 3.517004$$

$$4f(t_3) = 4f(1.3) = 4\sqrt{1 + \frac{1}{1.3^2} + (\ln 1.3)^2 + 2 \ln 1.3 + 1} \approx 7.138941$$

$$2f(t_4) = 2f(1.4) = 2\sqrt{1 + \frac{1}{1.4^2} + (\ln 1.4)^2 + 2 \ln 1.4 + 1} \approx 3.631177$$

$$4f(t_5) = 4f(1.5) = 4\sqrt{1 + \frac{1}{1.5^2} + (\ln 1.5)^2 + 2 \ln 1.5 + 1} \approx 7.397055$$

$$2f(t_6) = 2f(1.6) = 2\sqrt{1 + \frac{1}{1.6^2} + (\ln 1.6)^2 + 2 \ln 1.6 + 1} \approx 3.769104$$

$$4f(t_7) = 4f(1.7) = 4\sqrt{1 + \frac{1}{1.7^2} + (\ln 1.7)^2 + 2 \ln 1.7 + 1} \approx 7.682545$$

$$2f(t_8) = 2f(1.8) = 2\sqrt{1 + \frac{1}{1.8^2} + (\ln 1.8)^2 + 2 \ln 1.8 + 1} \approx 3.913928$$

$$4f(t_9) = 4f(1.9) = 4\sqrt{1 + \frac{1}{1.9^2} + (\ln 1.9)^2 + 2 \ln 1.9 + 1} \approx 7.972646$$

$$f(t_{10}) = f(2) = \sqrt{1 + \frac{1}{2^2} + (\ln 2)^2 + 2 \ln 2 + 1} \approx 2.028977$$

$$\int_1^2 \sqrt{1 + \frac{1}{t^2} + (\ln t)^2 + 2 \ln t + 1} \, dt \approx \frac{0.1}{3}(1.732051 + 6.958334 +$$
$$+ 3.517004 + 7.138941 + 3.631177 + 7.397055 + 3.769104 +$$
$$+ 7.682545 + 3.913928 + 7.972646 + 2.028977) \approx 1.858$$