Answer on Question #72037 - Math - Statistics and Probability

Question

If the probability is 0.60 that a divorce will remarry within 3 years, find the probability that of 5 divorces:

- a) 3 out of 5 divorces will remarry within 3 years.
- b) At least 3 will remarry within 3 years.
- c) At most 3 will remarry within 3 years.

Solution

In this problem we have the binomial distribution with the following parameters (see https://en.wikipedia.org/wiki/Binomial distribution):

$$n = 5$$
, $p = 0.6$, $q = 1 - p = 0.4$.

Let X be the random variable and denote the number out of 5 divorces will remarry within 3 years.

a)
$$P(X = 3) = {5 \choose 3} \cdot (0.6)^3 \cdot (0.4)^2 = \frac{5!}{3! \cdot 2!} \cdot 0.216 \cdot 0.16 = 10 \cdot 0.216 \cdot 0.16 = 0.3456.$$

b)
$$P(X \ge 3) = P(X = 3) + P(X = 4) + P(X = 5) = {5 \choose 3} \cdot (0.6)^3 \cdot (0.4)^2 + (0.6)^3 \cdot (0.6)^4 \cdot (0.6)$$

$$+ \binom{5}{4} \cdot (0.6)^4 \cdot (0.4)^1 + \binom{5}{5} \cdot (0.6)^5 \cdot (0.4)^0 = 0.3456 + 0.2592 + 0.07776 = 0.68256.$$

c)
$$P(X \le 3) = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) = {5 \choose 0} \cdot (0.6)^0 \cdot (0.4)^5 + (0.6)^0 \cdot (0.4)^$$

$$+ \binom{5}{1} \cdot (0.6)^1 \cdot (0.4)^4 + \binom{5}{2} \cdot (0.6)^2 \cdot (0.4)^3 + \binom{5}{3} \cdot (0.6)^3 \cdot (0.4)^2 =$$

$$= 0.01024 + 0.0768 + 0.2304 + 0.3456 = 0.66304.$$

Answer:

- a) 0.3456;
- **b)** 0.68256;
- c) 0.66304.