Answer on Question #71803 – Math – Statistics and Probability

Question

In Orange County, 51% of the adults are males. One adult is randomly selected for a survey involving credit card usage.

a) Find the prior probability that the selected person is a female.
b) It is later learned that the selected survey subject was smoking a cigar. Also, 9.5% of males smoke cigars, whereas 1.7% of females smoke cigars. Use this additional information to find the probability that the selected subject is a female.

Solution

Let's use the following notation:
M = male
$\overline{M}$ = female (or not male)
C = cigar smoker
$\overline{C}$ = not cigar smoker

a) A prior probability is an initial probability value originally obtained before any additional information is obtained.
The prior probability that the selected person is a female
$$P(\overline{M}) = 1 - P(M) = 1 - 0.51 = 0.49$$

b) Bayes' Theorem
The probability of event A, given that event B has subsequently occurred, is
$$P(A|B) = \frac{P(A)P(B|A)}{[P(A)P(B|A)] + [P(\overline{A})P(B|\overline{A})]}$$
Based on the additional information:
$P(M) = 0.51$
$P(\overline{M}) = 0.49$
$P(C|M) = 0.095$
$P(C|\overline{M}) = 0.017$
We can now apply Bayes' Theorem.
Then the probability that the selected subject is female using additional information that is later obtained
$$P(\overline{M}|C) = \frac{P(\overline{M})P(C|\overline{M})}{[P(\overline{M})P(C|\overline{M})] + [P(M)P(C|M)]}$$
$$P(\overline{M}|C) = \frac{0.49(0.017)}{[0.49(0.017)] + [0.51(0.095)]} \approx 0.1467$$