Answer on Question #71123 – Math – Calculus

Question

Prove that $lim_{x\to\infty}\left(\frac{11x+7}{8-5x}\right) = -\frac{11}{5}$ using the delta - epsilon definition of limits?

Solution

For given $\varepsilon > 0$, we need to choose δ, so that if $x > \delta$, then $\left|\frac{11x+7}{8-5x} + \frac{11}{5}\right| < \varepsilon$.

So, $\left|\frac{11x+7}{8-5x} + \frac{11}{5}\right| = \left|\frac{(11x+7)5}{(8-5x)5} + \frac{11(8-5x)}{5(8-5x)}\right| = \left|\frac{123}{40-25x}\right| < \left|-\frac{123}{25x}\right| < \frac{123}{25x} < \frac{123}{25\delta} < \varepsilon$

So, now for any $\varepsilon > 0$ there exists $\delta = \frac{123}{25\varepsilon}$ and satisfies $x > \delta$ and

$$\left|\frac{11x+7}{8-5x} + \frac{11}{5}\right| = \left|\frac{(11x+7)5}{(8-5x)5} + \frac{11(8-5x)}{5(8-5x)}\right| = \left|\frac{123}{40-25x}\right| < \left|-\frac{123}{25x}\right| < \frac{123}{25x} < \frac{123}{25\delta} =$$

$= \frac{123}{25} \times \frac{25\varepsilon}{123} = \varepsilon$, q.e.d.

Answer:

$lim_{x\to\infty}\left(\frac{11x+7}{8-5x}\right) = -\frac{11}{5}$