

## Answer on Question #70830– Math – Geometry

### Question

Calculate the arc length of catenary  $\gamma(t) = (t, \cosh t)$  starting at the point  $(0, 1)$ .

### Solution

The arc Length is given by

$$L = \int_{\alpha}^{\beta} \sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2} dt,$$

where

$$x(t) = t, \quad y(t) = \cosh t$$

$$\frac{dx}{dt} = 1$$

$$\frac{dy}{dt} = (\cosh t)' = \sinh t$$

$$\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2 = (\sinh t)^2 + 1 = \cosh^2 t.$$

Then

$$L = \int_0^t \sqrt{\cosh^2 t} dt = \int_0^t \cosh t dt = \sinh t \Big|_0^t = \sinh t - \sinh 0 = \sinh t$$

**Answer:**

The arc length of catenary is  $L = \sinh t$