Answer on Question #70822 – Math – Geometry

Question

1. Find parametrization of following level curve y^2-x^2=1.

Solution

The equation of the conjugate hyperbola in Cartesian coordinates is given by

$$\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$$

A parametrization of the conjugate hyperbola is

$$x=a\cdot\sinh t\ ,\ \ y=b\cdot\cosh t\ ,\qquad t\in R$$

(the hyperbolic identity $\cosh^2(t)-\sinh^2(t)=1$ was applied),

then the curve

$$y^2-x^2=1\ ,\qquad a=1,\qquad b=1$$

has the following parametrization:

$$x=\sinh t\ ,\ \ y=\cosh t\ ,\qquad t\in R$$

Answer: $x=\sinh t\ ,\ \ y=\cosh t\ ,\ t\in R$

Question

2. Find parametrization of following level curve x^2/4+y^2/9=1.

Solution

The equation of an ellipse in Cartesian coordinates is given by

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

A parametrization of ellipse curve is

$$x=a\cdot\cos t\ ,\ \ y=b\cdot\sin t\ ,\qquad 0\le t\le 2\pi$$

(the trigonometric identity $\cos^2(t)+\sin^2(t)=1$ was applied),

then the curve

$$\frac{x^2}{4}+\frac{y^2}{9}=1\ ,\qquad a=2,\qquad b=3$$

has the following parametrization:

$$x = 2 \cdot \cos t\,, \;\; y = 3 \cdot \sin t\,, \;\; 0 \le t \le 2\pi$$

Answer: $x = 2 \cdot \cos t\,, \;\; y = 3 \cdot \sin t, \;\; 0 \le t \le 2\pi.$