Answer on Question #70766 – Math – Statistics and Probability

Question

1. Suppose X and Y are independent continuous random variables. Show that $E[X|Y = y] = E[X]$ for all y.

Solution

$$E[X|Y = y] = |by\ definition| = \sum_i x_i P\{X = x_i|Y = y\} = \sum_i x_i P\{X = x_i\} = E[X].$$

Question

2. The joint density of X and Y is

$$f\,(x,y) = (y^2 - x^2)\ e^{-y}, \qquad 0 < y < \infty, \quad -y \le x \le y.$$

Show that $E[X|Y = y] = 0$.

Solution

$$E[X|Y = y] = \int_{-\infty}^{\infty} x \frac{f(x,y)}{f_Y(y)} dx.$$

$$f_Y(y) = \int_{-\infty}^{\infty} f(x,y)dx = \int_{-y}^{y} (y^2 - x^2)e^{-y}dx = xy^2e^{-y}|_{-y}^{y} - \frac{x^3e^{-y}}{3}|_{-y}^{y} = \frac{4}{3}e^{-y}y^3.$$

Hence,

$$E[X|Y = y] = \frac{3}{4}\int_{-\infty}^{\infty} \frac{x(y^2 - x^2)e^{-y}}{e^{-y}y^3}dx = \frac{3}{4y}\int_{-\infty}^{\infty} xdx - \frac{3}{4y^3}\int_{-\infty}^{\infty} x^3dx$$

$$= |the\ integral\ of\ the\ odd\ function\ in\ a\ symmetric\ boundary\ is\ zero| = 0.$$