Suppose a special type of small data processing firm is so specialized that some have difficulty making a profit in their first year of operation. The p.d.f that characterizes the proportion Y that make a profit is given by: $ky^4(1-y)^3$ and 0 elsewhere.

A. What is the value of k that renders the above a valid density function?
B. B.Find the probability that at most 50% of the firms make a profit in the first year.
C. C. Find the probability that at least 80% of the firms make a profit in the first year.

Solution.

A.

$$\int_0^1 ky^4(1-y)^3dy = k\int_0^1 (y^4-3y^5+3y^6-y^7)dy = k\left(\frac{y^5}{5}-\frac{3y^6}{6}+\frac{3y^7}{7}-\frac{y^8}{8}\right)\Bigg|_0^1 =$$

$$= \frac{k}{280} \Rightarrow k = 280.$$

B.

$$\int_0^{0,5} 280y^4(1-y)^3dy = 280\int_0^{0,5} (y^4-3y^5+3y^6-y^7)dy =$$

$$= 280\left(\frac{y^5}{5}-\frac{3y^6}{6}+\frac{3y^7}{7}-\frac{y^8}{8}\right)\Bigg|_0^{0,5} \cong 0.363.$$

C.

$$\int_{0,8}^{1} 280y^4(1-y)^3dy =$$

$$= 280\int_{0,8}^{1} (y^4-3y^5+3y^6-y^7)dy =$$

$$= 280\left(\frac{y^5}{5}-\frac{3y^6}{6}+\frac{3y^7}{7}-\frac{y^8}{8}\right)\Bigg|_{0,8}^{1} \cong 0,056.$$