Answer on Question #70312, Math / Calculus

An efficiency study of the workers at a factory shows that an average worker who comes to work at 8:00am will have produced $Q(t) = -t^3 + 9t^2 + 12t$ units t hours later. At what time during the morning is the worker performing most efficiently.

Solution.

The productivity is given as:

$$Q(t) = -t^3 + 9t^2 + 12t .$$

To find the optimum productivity we seek a critical point of $Q(t)$, and would expect to find a maxima.

Differentiating gives:

$$Q'(t) = -3t^2 + 18t + 12 .$$

At a critical point $Q'(t) = 0$:

$$-3t^2 + 18t + 12 = 0 ,$$

$$t^2 - 6t - 4 = 0 ,$$

$$t = \frac{6 \pm \sqrt{36+16}}{2} = \frac{6 \pm \sqrt{52}}{2} = \frac{6 \pm 2\sqrt{13}}{2} = 3 \pm \sqrt{13} .$$

Since $t > 0$, $t = 3 + \sqrt{13}$

We can do a second derivative test to verify this is a maximum;

$$Q''(t) = -6t + 18 .$$

If $t = 3 + \sqrt{13}$, then $Q''(t) = -6(3+\sqrt{13}) + 18 = -6\sqrt{13} < 0$. Since $Q''(t) < 0$, $t = 3 + \sqrt{13}$ is maximum.

Thus the maximum productivity occurs when $t = 3 + \sqrt{13} \approx 6.6056$, which correspond to a duration of 6 h 36 m. As $t = 0$ was 8 am then the optimum production time would be 2:36 pm.