

Answer on Question #70257 – Math – Differential Equations

Question

Solve the given differential equation by using appropriate substitution:

$$\frac{dy}{dx} = \frac{1 - x - y}{x + y}$$

Solution

$$\begin{aligned} \text{Let } g(x) = -x - y(x) \Rightarrow y(x) = -x - g(x) \Rightarrow -1 - \frac{dg(x)}{dx} &= \frac{1+g(x)}{-g(x)} \Rightarrow 1 + \frac{dg(x)}{dx} = \frac{1+g(x)}{g(x)} \Rightarrow \\ \Rightarrow 1 + \frac{dg(x)}{dx} &= \frac{1}{g(x)} + 1 \Rightarrow \frac{dg(x)}{dx} = \frac{1}{g(x)} \Rightarrow \frac{dg(x)}{dx} g(x) = 1 \Rightarrow \int g'(x)g(x)dx = \int 1dx \end{aligned}$$

$$\int g'(x)g(x)dx = \left[\begin{array}{l} u = g(x) \\ du = g'(x)dx \end{array} \right] = \int u du = \frac{u^2}{2} + c_1 = \frac{g^2(x)}{2} + c_1$$

$$\frac{g^2(x)}{2} + c_1 = x + c_2 \text{ or } \frac{g^2(x)}{2} = x + c \Rightarrow \frac{(x+y)^2}{2} = x + c$$

Answer:

$$\frac{(x+y)^2}{2} = x + c$$