Answer on Question #70245 - Math / Calculus

Question.
Sketch the graph of the function f defined by f(x) =x⁴+8x³, clearly giving all the properties used in it.

Solution
Domain $(-\infty, +\infty)$
1. Find the points of intersection with the axis of abscissas ox
$f(x) = x^4 + 8x^3 = 0$
$x^3(x + 8) = 0$
$x_{1,2,3} = 0$
$x + 8 = 0$
$x_4 = -8$
2. The intervals of increase and decreasing function
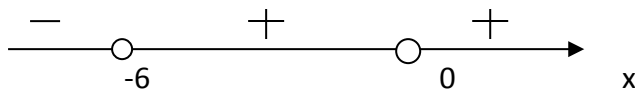$f'(x) > 0, f'(x) < 0$
$f'(x) = 4x^3 + 24x^2$
$4x^3 + 24x^2 = 0$
$4x^2(x + 6) = 0$
$x_{1,2} = 0$
$x + 6 = 0$
$x_3 = -6$

$$\underset{-6}{\underbrace{\qquad - \qquad}} \circ \underset{\ }{\underbrace{\qquad + \qquad}} \underset{0}{\circ} \underset{x}{\underbrace{\quad + \quad}} \rightarrow$$

Here the critical points
$x_1 = -6$ from "-" goes to "+" means minimum function $f(-6) = (-6)^4 + 8 \cdot (-6)^3 = -432$
Range $[-432, +\infty)$
$x_2 = 0$, at this point the function does not increase and does not decrease.
3. Find the intervals convexity and concavity of a function
$f''(x) > 0, f''(x) < 0$
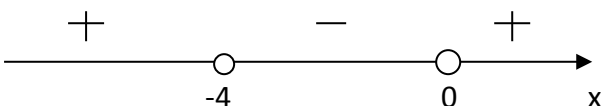$f'(x) = 4x^3 + 24x^2$
$f''(x) = 12x^2 + 48x$
$12x^2 + 48x = 0$
$12x(x + 4) = 0$
$x_1 = 0$
$x + 4 = 0$
$x_2 = -4$

$$\underset{\ }{\underbrace{\quad + \quad}} \underset{-4}{\circ} \underset{\ }{\underbrace{\quad - \quad}} \underset{0}{\circ} \underset{x}{\underbrace{\ + \ }} \rightarrow$$
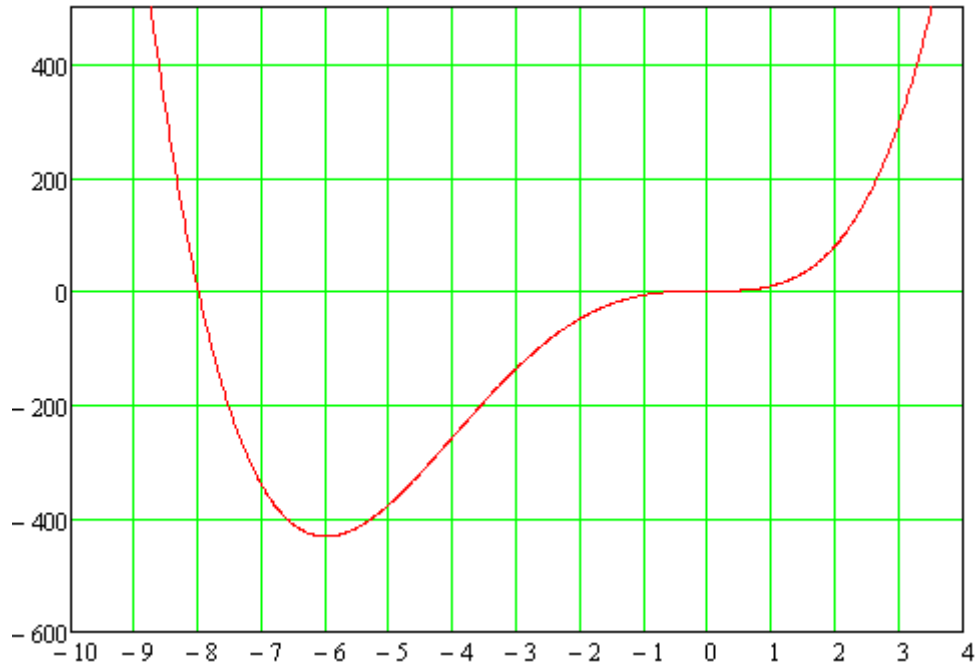
$f''(x) > 0, (-\infty, -4) \cup (0, +\infty)$ Concavity
$f''(x) < 0, (-4,0)$ Convexity

$x_1 = -4, x_2 = 0$ Inflection points

Get the following graph