Answer on Question #70095 – Math – Linear Algebra

Question

Let V = R^2
Define addition + on V by (x1, y1) + (x2, y2) = (x1 +x2, y1 +y2) and scalar multiplication · by r·(a,b) = (ra,0). Check whether V satisfies all the conditions for it to be a vector space over R with respect to these operations.

Solution

Addition operation:
1) (x1,y1)+(x2,y2)=(x1 +x2, y1 +y2)= (x2,y2)+(x1,y1) -commutativity
2) (x1,y1)+((x2,y2)+(x3,y3))= (x1,y1)+ (x3 +x2, y3 +y2)= (x1+x2 +x3, y1 +y2+y3)= ((x1,y1)+(x2,y2))+(x3,y3) -associativity
3)Zero element: (0,0): (x1,y1)+(0,0)=(x1,y1)
4)Inverse element: (x1,y1)+(-x1,-y1)=(0,0)

Scalar multiplication:
1)a(b(x,y))=a(bx,0)= (abx,0)=b(ax,0)=b(a(x,y)) – compatibility
2)Identity element: 1(x,y)=(x,0)≠(x,y) – the identity element does not exist for this operation.
3)a((x1,y1)+(x2,y2))=a(x1 +x2, y1 +y2)=(a(x1+x2),0)=a(x1,y1)+a(x2,y2) – distributivity law
4)(a+b)(x,y)=((a+b)x,0)=(ax,0)+(bx,0)=a(x,y)+b(x,y) – distributivity law.

Answer:
This space doesn't satisfy the scalar multiplication axiom (this operation does not have the identity element). So, it is not a vector space.