Answer on Question #70038 – Math – Calculus

Question

Evaluate $\vec{\nabla}(r^3\vec{r})$.

Solution

$$F(x,y,z) = r^3\vec{r} = \sqrt{(x^2+y^2+z^2)^3}\left(x\vec{\imath} + y\vec{\jmath} + z\vec{k}\right) =$$

$$= x\sqrt{(x^2+y^2+z^2)^3}\cdot\vec{\imath} + y\sqrt{(x^2+y^2+z^2)^3}\cdot\vec{\jmath} + z\sqrt{(x^2+y^2+z^2)^3}\cdot\vec{k} = F_1\vec{\imath} + F_2\vec{\jmath} + F_3\vec{k},$$

$$\vec{\nabla}(r^3\vec{r}) = \vec{\nabla}F(x,y,z) = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z} =$$

$$\frac{\partial}{\partial x}\left(x\sqrt{(x^2+y^2+z^2)^3}\right) + \frac{\partial}{\partial y}\left(y\sqrt{(x^2+y^2+z^2)^3}\right) + \frac{\partial}{\partial z}\left(z\sqrt{(x^2+y^2+z^2)^3}\right) =$$

$$= \left(\sqrt{(x^2+y^2+z^2)^3} + x\cdot\frac{3}{2}\sqrt{x^2+y^2+z^2}\cdot 2x\right) +$$

$$+ \left(\sqrt{(x^2+y^2+z^2)^3} + y\cdot\frac{3}{2}\sqrt{x^2+y^2+z^2}\cdot 2y\right) +$$

$$+ \left(\sqrt{(x^2+y^2+z^2)^3} + z\cdot\frac{3}{2}\sqrt{x^2+y^2+z^2}\cdot 2z\right) =$$

$$= 3\sqrt{(x^2+y^2+z^2)^3} + \sqrt{x^2+y^2+z^2}(3x^2+3y^2+3z^2) =$$

$$= 3\sqrt{(x^2+y^2+z^2)^3} + 3\sqrt{(x^2+y^2+z^2)^3} = 6\sqrt{(x^2+y^2+z^2)^3} = 6r^3.$$

Answer: $6r^3$.