

Answer on Question #70036 – Math – Calculus

Question

Obtain the directional derivative for a scalar field $\phi(x, y, z) = 3(x^2)y - (y^3)(z^2)$ at the point $(1, -2, -1)$ in the direction $\hat{i} + \hat{j} + \hat{k}$.

Solution

$$\phi(x, y, z) = 3(x^2)y - (y^3)(z^2)$$

$$\nabla\phi = \left(\frac{\partial\phi}{\partial x}, \frac{\partial\phi}{\partial y}, \frac{\partial\phi}{\partial z}\right) = (6xy, 3x^2 - 3y^2z^2, -2y^3z)$$

$$\nabla\phi(1, -2, -1) = (6(1)(-2), 3(1)^2 - 3(-2)^2(-1)^2, -2(-2)^3(-1))$$

$$\nabla\phi(1, -2, -1) = (-12, -9, -16)$$

$$|\hat{i} + \hat{j} + \hat{k}| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$$

The directional derivative is

$$\frac{\partial\phi}{\partial l}(1, -2, -1) = (-12, -9, -16) \frac{(1, 1, 1)}{\sqrt{3}} = \frac{1}{\sqrt{3}}(-12(1) - 9(1) - 16(1)) = -\frac{37}{\sqrt{3}}$$

Answer: $-\frac{37}{\sqrt{3}}$.