Answer on Question #70024 – Math – Calculus

Question

Use the mean value theorem to show that x < sin^{-1}x , for x > 0

Solution

According to the mean-value theorem, if a function f(x) is continuous on a segment $[a,b]$, $b > a$, and is differentiable on the interval (a,b) then there exists some c belonging to the open interval such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

Let

$$a = 0, \qquad b = x, \qquad f(x) = \sin^{-1} x.$$

Then

$$f'(x) = \frac{1}{\sqrt{1 - x^2}}, \qquad 0 \leq x < 1.$$

It follows from the mean-value theorem that for some $c \in (0,1)$

$$\frac{1}{\sqrt{1 - c^2}} = \frac{\sin^{-1} x - \sin^{-1} 0}{x - 0} = \frac{\sin^{-1} x}{x}.$$

Therefore,

$$x = \sin^{-1} x \cdot \sqrt{1 - c^2}, \qquad 0 < c < 1,$$

so

$$x < \sin^{-1} x$$

for $x > 0$.