Answer on Question #70011 – Math – Calculus

Question

Sketch the graph of the function f defined by $f(x) = x^4 + 8x^3$, clearly giving all the properties used in it.

Solution

$$f(x) = x^4 + 8x^3$$

This is the fourth degree polynomial or quartic polynomial.
Quartics have these characteristics:
Zero to four roots
One, two or three extrema
Zero, one or two inflection points
No general symmetry.
It takes five points or five pieces of information to describe a quartic function.
Roots are solvable by radicals. (Very advanced and complicated.)

End behavior:

Degree	Leading coefficient	End behavior of the function
$n = 4$, even	$a_4 = 1 > 0$, positive	$f(x) \to +\infty$, as $x \to -\infty$ $f(x) \to +\infty$, as $x \to +\infty$

Find the roots
$f(x) = 0 \Rightarrow x^4 + 8x^3 = 0$
$x^3(x + 8) = 0$
$x = -8$ or $x = 0$
Find the first derivative
$\frac{df}{dx} = f'(x) = (x^4 + 8x^3)' = 4x^3 + 8(3x^2) = 4x^3 + 24x^2$
Find the critical point(s)
$f'(x) = 0 \Rightarrow 4x^3 + 24x^2 = 0$
$4x^2(x + 6) = 0$
$x = -6$ or $x = 0$
The First Derivative Test
If $x < -6$, then $f'(x) < 0$, $f(x)$ is decreasing.
If $-6 < x < 0$, then $f'(x) > 0$, $f(x)$ is increasing.
If $x > 0$, then $f'(x) > 0$, $f(x)$ is increasing.
Then f has a relative minimum value at $x = -6$; f has no extrema at $x = 0$.

$$f(-6) = (-6)^4 + 8(-6)^3 = -432$$

Find the second derivative
$f''(x) = (4x^3 + 24x^2)' = 4(3x^2) + 24(2x) = 12x^2 + 48x$
$f''(x) = 0 \Rightarrow 12x^2 + 48x = 0$
$12x(x + 4) = 0$

$x = -4$ or $x = 0$

If $x < -4$, then $f''(x) > 0$, the graph of f is concave upward.

If $-4 < x < 0$, then $f''(x) < 0$, the graph of f is concave downward.

If $x > 0$, then $f''(x) > 0$, the graph of f is concave upward.

$$f(-4) = (-4)^4 + 8(-4)^3 = -256$$

The inflection points: $(-4, -256), (0, 0)$.

Sketch the graph of the function f