

**Answer on Question #69887 – Math – Differential Equations
Question**

The orthogonal trajectories of all the parabolas with vertices at the origin and foci on the x-axis is

$$x^2 + 2y^2 = c^2$$

True or false?

Solution

A procedure for finding a family of orthogonal trajectories $G(x, y, K) = 0$ for a given family of curves $F(x, y, C) = 0$ is as follows:

1. Determine the differential equation for the given family $F(x, y, C) = 0$.
2. Replace y' in that equation by $-1/y'$; the resulting equation is the differential equation for the family of orthogonal trajectories.
3. Find the general solution of the new differential equation. This is the family of orthogonal trajectories.

1. We have the family of parabolas

$$x = ay^2, \quad a \text{ is real constant}$$

Differentiate both sides with respect to x

$$1 = 2a yy'$$
$$y' = \frac{1}{2ay}, \quad a = \frac{x}{y^2}$$

The differential equation for the given family can be written as

$$y' = \frac{1}{2ay} = \frac{y^2}{2xy} = \frac{y}{2x}$$

2. Replace y' in the equation by $-1/y'$

$$-\frac{1}{y'} = \frac{y}{2x}$$

3. Find the general solution of the new differential equation

$$-\frac{1}{y'} = \frac{y}{2x}$$
$$y' = -2\frac{x}{y}$$
$$y dy = -2x dx$$
$$\int y dy = -\int 2x dx$$
$$\frac{1}{2}y^2 = -2\left(\frac{1}{2}\right)x^2 + \frac{1}{2}c^2$$

Hence, the equation of the orthogonal trajectories is

$$2x^2 + y^2 = c^2$$

Answer: false.