Answer on Question #69804 – Math – Calculus

Question

Expand $\sin x \cdot \cosh x$ in ascending powers of x up to x^5.

Solution

Method I

$f(x) = \sin x \cdot \cosh x$
$f(0) = 0$
$f'(x) = (\sin x \cdot \cosh x)' = (\sin x)'(\cosh x) + (\sin x)(\cosh x)' =$
$= \cos x \cdot \cosh x + \sin x \cdot \sinh x$
$f'(0) = 1 \cdot 1 + 0 = 1$
$f''(x) = (\cos x \cdot \cosh x + \sin x \cdot \sinh x)' =$
$= (\cos x)'(\cosh x) + (\cos x)(\cosh x)' + (\sin x)'(\sinh x) + (\sin x)(\sinh x)' =$
$= -\sin x \cdot \cosh x + \cos x \cdot \sinh x + \cos x \cdot \sinh x + \sin x \cdot \cosh x =$
$= 2\cos x \cdot \sinh x$
$f''(0) = 0$
$f'''(x) = (2\cos x \cdot \sinh x)' = 2(\cos x)'(\sinh x) + 2(\cos x)(\sinh x)' =$
$= -2\sin x \cdot \sinh x + 2\cos x \cdot \cosh x$
$f'''(0) = 0 + 2 \cdot 1 \cdot 1 = 2$
$f^{\text{iv}}(x) = (-2\sin x \cdot \sinh x + 2\cos x \cdot \cosh x)' =$
$= -2(\sin x)'(\sinh x) - 2(\sin x)(\sinh x)' +$
$+2(\cos x)'(\cosh x) + 2(\cos x)(\cosh x)' =$
$= -2\cos x \cdot \sinh x - 2\sin x \cdot \cosh x - 2\sin x \cdot \cosh x + 2\cos x \cdot \sinh x =$
$= -4\sin x \cdot \cosh x$
$f^{\text{iv}}(0) = 0$
$f^{\text{v}}(x) = (-4\sin x \cdot \cosh x)' = -4(\sin x)'(\cosh x) - 4(\sin x)(\cosh x)' =$
$= -4\cos x \cdot \cosh x - 4\sin x \cdot \sinh x$
$f^{\text{v}}(0) = -4 \cdot 1 \cdot 1 - 0 = -4$
By Maclaurin's Theorem we have

$$f(x) = f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + \frac{x^3}{3!}f'''(0) + \frac{x^4}{4!}f^{\text{iv}}(0) + \frac{x^5}{5!}f^{\text{v}}(0) + \cdots$$

Substituting all the values we get

$$f(x) = \sin x \cdot \cosh x = 0 + x + 0 + \frac{x^3}{3!}(2) + 0 + \frac{x^5}{5!}(-4) + \cdots =$$

$$= x + \frac{x^3}{3} - \frac{x^5}{30} + \cdots$$

Method II

Use Standard Expansions

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

$$\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \cdots$$

Then

$$\sin x \cdot \cosh x = \left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots\right)\left(1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \cdots\right) =$$

$$= x - \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^3}{2!} - \frac{x^5}{2!\,3!} + \frac{x^5}{4!} + \cdots =$$

$$= x + \frac{x^3}{6}(-1 + 3) + \frac{x^5}{120}(1 - 10 + 5) + \cdots =$$

$$= x + \frac{x^3}{3} - \frac{x^5}{30} + \cdots$$

Answer: $\sin x \cdot \cosh x = x + \frac{x^3}{3} - \frac{x^5}{30} + \cdots$