Question

Confirm that f and g are inverses by showing that f(g(x)) = x and g(f(x)) = x.
f(x) = the quantity x minus seven divided by the quantity x plus three. and g(x) = quantity negative three x minus seven divided by quantity x minus one.

Solution

We have the following definitions:

$$f(x) = \frac{x-7}{x+3}, \quad g(x) = \frac{-3x-7}{x-1}.$$

Let us make required substitutions:

$$f\big(g(x)\big) = \frac{g(x)-7}{g(x)+3} = \frac{\dfrac{-3x-7}{x-1}-7}{\dfrac{-3x-7}{x-1}+3} = \frac{-3x-7-7(x-1)}{-3x-7+3(x-1)} = \frac{-10x}{-10} = x,$$

$$g\big(f(x)\big) = \frac{-3f(x)-7}{f(x)-1} = \frac{-3\dfrac{x-7}{x+3}-7}{\dfrac{x-7}{x+3}-1} = \frac{-3(x-7)-7(x+3)}{x-7-(x+3)} = \frac{-10x}{-10} = x.$$