Answer to Question #69654 – Math – Differential Equations

Question

One hundred grams of cane sugar in water are being converted into dextrose at a rate which is proportional to the amount unconverted. Find the differential equation expressing the rate of conversion after T minutes.

Solution

m_s – mass of sugar at the current moment of time

m_D – mass of dextrose at the current moment f time

$m_{s_0} = 100g$ – mass of sugar at the beginning

T – the time that had passed

C – proportion coefficient

The fact that the sugar is converted in to dextrose at a rate which is proportional to the amount of sugar unconverted in terms of differentiates means

$$dm_D = Cm_s dt$$

where

$$m_s = m_{s_0} - m_D$$

because sucrose is converted somehow to dextrose and we can in general neglect the mas of one molecule of water per a molecule of sucrose (thus finally two molecules of dextrose).

So

$$dm_D = C\left(m_{s_0} - m_D\right)dt$$

Solving it we get

$$\ln\left(m_{s_0} - m_D\right) = -Ct + K$$

$$m_D = m_{s_0} - e^{K-Ct} = m_{s_0} - \frac{e^K}{e^{Ct}}$$

When t=0 there is no dextrose so

$$0 = m_{s_0} - \frac{e^K}{e^{-C*0}} \rightarrow e^K = m_{s_0}$$

and therefore

$$m_D = m_{s_0}\left(1 - \frac{1}{e^{Ct}}\right) = m_{s_0}(1 - e^{-Ct})$$

Thus, the rate of conversion after T minutes:

$$dm_D = C\left(m_{s_0} - m_D\right)dt = Cm_{s_0}e^{-CT}dt$$

But we does not take the deferential equation! It should be treated in the terms of delta

$$\Delta m_D = Cm_{s_0}e^{-CT}\Delta t$$

only when $T \gg \Delta t$.

The differential equation expressing the rate of conversion after T minutes will look like

$$dm_D = C\left(m_{s_0} - Cm_{s_0}e^{-CT} - m_D\right)dt = C\left(m_{s_0}(1 - Ce^{-CT}) - m_D\right)dt$$

In the last formula (after integrating) time should be counted starting from moment *T*.

Answer provided by https://www.AssignmentExpert.com