Answer on Question #69649 – Math – Differential Equations

Question

Find the complete solution of

(D4−8D2+16)y=0

Solution

The equation

$$(D^4 - 8D^2 + 16)y(x) = 0$$

is a linear homogeneous ordinary differential equations with constant coefficients.

To solve this equation we set

$$y(x) = e^{kx}.$$

Then

$$D^4 y(x) = k^4 e^{kx}, \quad D^2 y(x) = k^2 e^{kx}.$$

Therefore we obtain the characteristic equation is of the form

$$k^4 - 8k^2 + 16 = 0$$

or

$$(k^2 - 4)^2 = 0.$$

The roots of the characteristic equation are

$$k_1 = 2, k_2 = 2, k_3 = -2, k_4 = -2.$$

Finally, the complete solution of the differential equation is given by

$$y(x) = C_1 e^{2x} + C_2 x e^{2x} + C_3 e^{-2x} + C_4 x e^{-2x},$$

where C_1, C_2, C_3, C_4 are arbitrary real constants.

Answer: $y(x) = C_1 e^{2x} + C_2 x e^{2x} + C_3 e^{-2x} + C_4 x e^{-2x}$.