Answer on Question #69644 - Math - Differential Equations

Find the particular integral of $\frac{dy}{dx} + y = \cos 3x$.

Solution

The corresponding homogenous equation is $\frac{dy}{dx} + y = 0$. Its characteristic equation is $\mu + 1 = 0$ with the root $\mu = -1$. With respect to it and considering the right part of the equation is $\cos 3x$, the particular integral is

$$y_p(x) = \alpha \cos 3x + \beta \sin 3x,$$

where α and β are the coefficients to be evaluated.

The derivative of the particular integral is

$$\frac{dy_p}{dx} = -3\alpha \sin 3x + 3\beta \cos 3x,$$

Substituting the particular integral and its derivative into the equation we get

$$\frac{dy_p}{dx} + y_p = (-3\alpha \sin 3x + 3\beta \cos 3x) + (\alpha \cos 3x + \beta \sin 3x) =$$
$$= (\alpha + 3\beta) \cos 3x + (\beta - 3\alpha) \sin 3x = \cos 3x.$$

To evaluate the unknown coefficients we get the system

$$\begin{matrix} \alpha + 3\beta = 1 \\ \beta - 3\alpha = 0 \end{matrix}.$$

From the second equation we have $\beta = 3\alpha$, thereby from the first equation we have $\alpha + 9\alpha = 1$. The solution is $\alpha = \frac{1}{10}$, thereby $\beta = 3 \cdot \frac{1}{10} = \frac{3}{10}$.

The particular integral is $y_p(x) = \frac{1}{10}\cos 3x + \frac{3}{10}\sin 3x$.

Answer: $y_p(x) = \frac{1}{10}\cos 3x + \frac{3}{10}\sin 3x$.