Answer on Question #69640 – Math – Differential Equation

Question

Solve the equation dt2dx2−4t=0

Solution

Case 1. If the differential equation is separable, then

$$\left(\frac{dt(x)}{dx}\right)^2 - 4t(x) = 0$$

Solve for $\frac{dt(x)}{dx}$

$\frac{dt(x)}{dx} = -2\sqrt{t(x)}$ or $\frac{dt(x)}{dx} = 2\sqrt{t(x)}$

For $\frac{dt(x)}{dx} = -2\sqrt{t(x)}$

Divide both sides by $2\sqrt{t(x)}$ and multiply by dx:

$$\frac{dt(x)}{2\sqrt{t(x)}} = -dx$$

Integrate both sides:

$\int \frac{dt(x)}{2\sqrt{t(x)}} = -\int dx + c_1$, where c_1 is an integration constant.

Solve for $t(x)$:

$$t(x) = (-x + c_1)^2$$

For $\frac{dt(x)}{dx} = 2\sqrt{t(x)}$

Divide both sides by: $2\sqrt{t(x)}$ and multiply by dx:

$$\frac{dt(x)}{2\sqrt{t(x)}} = dx$$

Integrate both sides:

$\int \frac{dt(x)}{2\sqrt{t(x)}} dx = \int dx + c_1$, where c_1 is an integration constant.

Solve for t(x):

$$t(x) = (x + c_1)^2$$

Answer: $t(x) = (-x + c_1)^2$ or $t(x) = (x + c_1)^2$.

Case 2. If it is a second order linear differential equation

$$\frac{d^2t}{dx^2} - 4t = 0,$$

then

$$\lambda^2 - 4 = 0,$$
$$\lambda_1 = 2, \lambda_2 = -2,$$
$$t(x) = c_1e^{\lambda_1 x} + c_2e^{\lambda_2 x} = c_1e^{-2x} + c_2e^{2x},$$

where c_1 and c_2 are arbitrary real constants.

Answer: $t(x) = c_1e^{-2x} + c_2e^{2x}$.