Answer on Question #69637 – Math – Differential Equations

Question

$$(x^2 + y^2)dx - 2xydy = 0$$

Solution

$$(x^2 + y^2)dx - 2xydy = 0$$

$$x^2 + y^2 - 2xyy' = 0$$

$$yy' = \frac{x^2 + y^2}{2x}$$

$$y' = \frac{x^2 + y^2}{2xy}$$

$$y' = \frac{x}{2y} + \frac{y}{2x}$$

$$y' - \frac{y}{2x} = \frac{x}{2} \cdot \frac{1}{y}$$

Method 1

$$yy' - \frac{1}{2x}y^2 = \frac{x}{2}$$

$$z = y^2 \quad (1)$$

$$\frac{1}{2}z' - \frac{1}{2x}z = \frac{x}{2}$$

$$z' - \frac{1}{x}z = x \quad (2)$$

The integrating factor of the differential equation (2) is

$$e^{\int -\frac{1}{x}dx} = e^{-\ln x} = \frac{1}{x}$$

Multiplying both sides of (2) by $\frac{1}{x}$

$$\frac{1}{x}z' - \frac{1}{x^2}z = 1$$

$$\left(\frac{1}{x} \cdot z\right)' = 1$$

Integrating both sides with respect to x

$$\frac{z}{x} = x + C$$

$$z = x(x + C)$$

Get back to substitution (1) and replace the variable in the previous equality

$$y^2 = x(x + C)$$

$$y = \pm\sqrt{x \cdot (x + C)}$$

Method 2

$$y' - \frac{y}{2x} = \frac{x}{2}y^{-1}$$

$y' + P(x)y = Q(x)y^n,\ n \neq 1,$ is the Bernoulli differential equation:

$z = \frac{1}{y^{n-1}}$,

$z = e^{-\int P_1(x)dx} \cdot \left(\int Q_1(x) \cdot e^{\int P_1(x)dx}\, dx + C\right),$
where

$Q_1(x) = -(n-1)Q(x),\ P_1(x) = -(n-1)P(x),$

$P(x) = -\frac{1}{2x},\ Q(x) = \frac{x}{2},\ n = -1,$

$P_1(x) = -\frac{1}{x}$, $Q_1(x) = x,$

$z = e^{-\int\left(-\frac{1}{x}\right)dx} \cdot \left(\int x \cdot e^{\int -\frac{1}{x}dx}\, dx + C\right),$

$e^{-\int\left(-\frac{1}{x}\right)dx} = e^{lnx} = x,$

$\int x \cdot e^{\int -\frac{1}{x}dx}\, dx = \int x \cdot e^{ln\frac{1}{x}}\, dx = \int x \cdot \frac{1}{x} dx = x,$

$z = x \cdot (x + C),$

$z = \frac{1}{y^{-2}},\ y^2 = z,$

$y^2 = x \cdot (x + C),$

$y = \pm\sqrt{x \cdot (x + C)}.$

Answer: $y = \pm\sqrt{x \cdot (x + C)}.$