Answer on Question #69634 – Math – Differential Equations

Question

Solve the differential equation

xdy/dx+y=x3

Solution

Divide both sides of the linear differential equation

$$xy' + y = x^3$$

by x and obtain

$$y' + \frac{y}{x} = x^2$$

Let

$$y = UV. \qquad \textbf{(1)}$$

Then

$$y' = U'V + V'U$$

$$U'V + V'U + \frac{UV}{x} = x^2$$

$$U'V + U\left(V' + \frac{V}{x}\right) = x^2.$$

If

$$V' + \frac{V}{x} = 0, \quad \textbf{(2)}$$

then

$$U'V + U \cdot 0 = x^2,$$

$$U'V = x^2. \quad \textbf{(3)}$$

Solving the equation (2)

$$V' + \frac{V}{x} = 0,$$

$$\frac{dV}{dx} = -\frac{V}{x},$$

$$\int \frac{dV}{V} = -\int \frac{dx}{x},$$

$$ln\, V = ln\frac{1}{x} + ln\, C.$$

Let $ln\, C = 0$, then

$$V = \frac{1}{x}. \qquad \textbf{(4)}$$

Substituting (4) into equation (3)

$$U'V = x^2$$

one gets

$$U'\frac{1}{x} = x^2,$$
$$U' = x^3,$$
$$U = \int x^3 dx,$$
$$U = \frac{x^4}{4} + C, \quad \textbf{(5)}$$

where C is an integration constant.
Substituting (4) and (5) into (1) one gets the solution of the initial differential equation:

$$y = UV \Rightarrow y = \left(\frac{x^4}{4} + C\right) \cdot \frac{1}{x},$$
$$y = \frac{x^3}{4} + \frac{C}{x}.$$

Answer: $y = \frac{x^3}{4} + \frac{C}{x}.$