Question

y'=(x^2+3x+2)/(y-2), y(2)=1

Solution

$$\frac{dy}{dx} = \frac{x^2 + 3x + 2}{y - 2}$$

$$(y - 2)dy = (x^2 + 3x + 2)dx$$

$$\int (y - 2)dy = \int (x^2 + 3x + 2)dx$$

$$\frac{y^2}{2} - 2y = \frac{x^3}{3} + \frac{3}{2}x^2 + 2x + C$$

If $y(2)=1$, then

$$\frac{1}{2} - 2 = \frac{2^3}{3} + \frac{3}{2}2^2 + 2 \cdot 2 + C$$

$$C = -\frac{85}{6}$$

And the implicit solution of the differential equation is

$$\frac{y^2}{2} - 2y = \frac{x^3}{3} + \frac{3}{2}x^2 + 2x - \frac{85}{6}$$

Answer: $\frac{y^2}{2} - 2y = \frac{x^3}{3} + \frac{3}{2}x^2 + 2x - \frac{85}{6}.$