Answer on Question #69631 – Math – Differential Equation

Question

Find the implicit solution of $y' = 2x + \frac{y}{5y^4} + 1, \ y(2) = 1$.

Solution

We write the equation as

$$\frac{dy}{dx} = 2x + \frac{y}{5y^4} + 1 \rightarrow dy - \left(2x + \frac{1}{5y^3} + 1\right)dx = 0$$

We represent the equation as

$$P(x,y)dx + Q(x,y)dy = 0.$$

Here

$$\frac{\partial P}{\partial y} = -\frac{3}{5y^4} \text{ and } \frac{\partial Q}{\partial x} = 0, \ \frac{\partial P}{\partial y} \neq \frac{\partial Q}{\partial x}.$$

Consider the quantities

$$\frac{1}{Q}\left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\right) = -\frac{3}{5y^4} \neq f(x) \text{ and } -\frac{1}{P}\left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\right) = \frac{1}{2x + \frac{1}{5y^3} + 1} \cdot \frac{3}{5y^4} \neq g(y)$$

The equation cannot be reduced to the total differential. Given that the equation is not homogeneous there are no methods for finding an exact solution.