Answer on Question #69214 – Math – Calculus

Question

1. The polynomials $ax^3 + 3x^2 - 13 \ \ and \ \ 2x^3 - 5x + a$ are divided by $(x + 2)$. If the remainder in each case is the same, find the value of a.

Solution

$$remainder\ of\ \frac{ax^3 + 3x^2 - 13}{x + 2} = a \cdot (-2)^3 + 3 \cdot 2^2 - 13$$

$$remainder\ of\ \frac{2x^3 - 5x + a}{x + 2} = 2 \cdot (-2)^3 - 5 \cdot (-2) + a$$

$$-8a + 12 - 13 = -16 + 10 + a$$

$$9a = 5$$
$$a = \frac{5}{9}$$

Answer: $a = \frac{5}{9}$.

Question

2. What must be subtracted from $4x^4 - 2x^3 - 6x^2 + x - 5$ so that the result is exactly divisible by $2x^2 + x - 1$? Use long division method.

Solution

$$\frac{4x^4 - 2x^3 - 6x^2 + x - 5}{2x^2 + x - 1} = 2x^2 - 2x - 1 - \frac{6}{2x^2 + x - 1}$$

$$4x^4 - 2x^3 - 6x^2 + x - 5 = (2x^2 - 2x - 1)(2x^2 + x - 1) - 6$$

$$(4x^4 - 2x^3 - 6x^2 + x - 5) + 6 = (2x^2 - 2x - 1)(2x^2 + x - 1)$$

$$(4x^4 - 2x^3 - 6x^2 + x - 5) - (-6) = (2x^2 - 2x - 1)(2x^2 + x - 1)$$

Answer: -6.

Question

3. What must be added to $x^4 + 2x^3 - 2x^2 + x - 1$ so that the result is exactly divisible by $x^2 + 2x - 3$? Use long division method.

Solution

$$\frac{x^4 + 2x^3 - 2x^2 + x - 1}{x^2 + 2x - 3} = x^2 + 1 + \frac{2 - x}{x^2 + 2x - 3}$$

$$x^4 + 2x^3 - 2x^2 + x - 1 = (x^2 + 1)(x^2 + 2x - 3) + 2 - x$$

$$(x^4 + 2x^3 - 2x^2 + x - 1) - (2 - x) = (x^2 + 1)(x^2 + 2x - 3)$$

$$(x^4 + 2x^3 - 2x^2 + x - 1) + x - 2 = (x^2 + 1)(x^2 + 2x - 3)$$

Answer: $x - 2$.

Question

4. If $a + b = 1$, then prove that $a^3 + b^3 + 3ab = 1$.

Solution

If $a + b = 1$, then $a^2 + 2ab + b^2 = 1$, $(a + b)^2 = 1$.

Consider

$a^3 + b^3 = (a + b)(a^2 - ab + b^2) = a^2 - ab + b^2 = a^2 + 2ab + b^2 - ab - 2ab = 1 - 3ab$,

hence

$$a^3 + b^3 + 3ab = 1$$

Question

5. Factorise:

a) $2\sqrt{2}a^3 + 16\sqrt{2}b^3 + c^3 - 12abc$

Answer: $2\sqrt{2}(a + 2b)(a^2 - 2ab + 4b^2) + c(c^2 - 12ab)$

b) $2\sqrt{2}x^3 + 3\sqrt{3}y^3 + \sqrt{5}(5 - 5\sqrt{6}xy)$

Answer: $2\sqrt{2}x^3 + 3\sqrt{3}y^3 + 5\sqrt{5}\left(1 - \sqrt{6}xy\right)$.

Question

6. Factorise:

$$\frac{a}{b}x^2 + \left(\frac{a}{b} + \frac{c}{d}\right)x \ + \frac{c}{d};\{\ b\ not\ equal\ to\ 0, d = 0\ \}$$

Solution

$$\frac{a}{b}x^2 + \left(\frac{a}{b} + \frac{c}{d}\right)x \ + \frac{c}{d} = \frac{a}{b}x(x+1) + \frac{c}{d}(x+1) = (x+1)\left(\frac{a}{b}x + \frac{c}{d}\right)$$

Answer: $(x+1)\left(\frac{a}{b}x + \frac{c}{d}\right)$.

Question

7. Factorise: $(ax\ -\ by)^3 + (by\ -\ cz)^3 + (cz\ -\ ax)^3$

Solution

$$(ax - by)^3 + (by - cz)^3 + (cz - ax)^3 =$$

$$= (ax - cz)((ax\ -\ by)^2 - (ax\ -\ by)(by\ -\ cz) + (by\ -\ cz)^2) + (cz\ -\ ax)^3$$

Answer: $(ax - cz)((ax\ -\ by)^2 - (ax\ -\ by)(by\ -\ cz) + (by\ -\ cz)^2 - (ax - cz)^2)$.