Answer on Question #69190 – Math – Statistics and Probability

Question

A shipment of 100 digital voice recorders contains 25 that are defective. If 10 of them are randomly chosen for inspection what is the probability that 2 of the 10 will be defective by using the Hypergeometric distribution by approximating the hypergeometric probability by the binomial probability.

Solution

Sampling without replacement means that we return no voice recorder to the box. This problem deals with a distribution called the hypergeometric distribution.
Consider all the possibilities as equally likely, there are a defective voice recorders, it follows that for sampling without replacement the probability of getting x successes (defective) in n trials is

$$h(x; n, a, N) = \frac{\binom{a}{x}\binom{N-a}{n-x}}{\binom{N}{n}} \text{ for } x = 0, 1, \dots, a$$

Note, when n is small as compared to N, less than $N/10$, the composition of the lot is not seriously affected by drawing the sample without replacement, and the binomial distribution with the parameters n and $p = a/N$ will yield a good approximation.
The hypergeometric distribution with large N approaches the binomial distribution.
We have that

$$x = 2, n = 10, a = 25, N = 100$$

a) The formula for the hypergeometric distribution is given by

$$h(2; 10, 25, 100) = \frac{\binom{25}{2}\binom{100-25}{10-2}}{\binom{100}{10}} = \frac{\binom{25}{2}\binom{75}{8}}{\binom{100}{10}} =$$

$$= \frac{25!\,75!\,10!\,(100-10)!}{2!\,(25-2)!\,8!\,(75-8)!\,100!} =$$

$$= \frac{25(24)(75)(74)(73)(72)(71)(70)(69)(68)(10)(9)}{2(100)(99)(98)(97)(96)(95)(94)(93)(92)(91)} = 0.2924$$

b) If

$$n = 10 = \frac{100}{10} = \frac{N}{10},\ p = \frac{25}{100} = 0.25,$$

then the formula for the binomial distribution as an approximation is

$$b(2; 10, 0.25) = \binom{10}{2}(0.25)^2(1-0.25)^{10-2} = \frac{10!}{2!\,(10-2)!}(0.25)^2(0.75)^8 =$$

$$= 0.2816.$$

Observe that the difference between the two values is

$$0.2924 - 0.2816 = 0.0108$$

In general, it can be shown that $h(x; n, a, N)$ approaches $b(x; n, p)$ with $p = a/N$ when N approaches infinity, the binomial distribution is used as an approximation to the hypergeometric distribution if $n \leq N/10$.