Answer on Question #69179 – Math – Calculus

Question

Write a polynomial function of minimum degree with real coefficients whose zeros include those listed. Write the polynomial in a standard form. 8, -14, and 3 + 9i.

Solution

If some number a is a zero of the given polynomial, then $(x-a)$ is a factor of this polynomial. Thus $(x-8)$ and $(x+14)$ are factors of the required polynomial.

The complex conjugate root theorem states that if p is a polynomial in one variable with real coefficients, and $a+ib$ is a root of p (a and b are real numbers), then its complex conjugate $a-ib$ is also a root of p. Thus, if the complex number $(3+9i)$ is a zero of the polynomial, then, in order that the coefficients were real, its conjugate $(3-9i)$ also must be a zero of this polynomial.

Therefore, for the required polynomial function we get

$$p(x)=(x-8)(x+14)\big(x-(3+9i)\big)\big(x-(3-9i)\big)=$$

$$=(x^2+14x-8x-112)\big(x^2-(3+9i)x-(3-9i)x+(3+9i)(3-9i)\big)=$$

$$=(x^2+6x-112)(x^2-6x+90)=$$

$$=x^4-6x^3+90x^2+6x^3-36x^2+540x-112x^2+672x-10080=$$

$$=x^4-58x^2+1212x-10080$$

Answer: the required polynomial function is

$$p(x)=x^4-58x^2+1212x-10080.$$