Answer on question #69109 – Math – Real Analysis

Question

Show that the set *B* = {x | x^(2) > 2} is non-empty and bounded below. Is it bounded above? Justify

Solution

The set $B = \{\mathrm{x} \mid \mathrm{x}\hat{}(2) > 2\}$ contains elements from the interval $(\sqrt{2}; +\infty)$, hence it is not empty.

This set is not bounded below because the statement $x^2 > 2$ means $(x < -\sqrt{2})$ or $(x > \sqrt{2})$, hence *B* is not bounded below because $\inf\limits_{x \in B} B = -\infty$.

The same reason gives that *B* is not bounded above because $\sup\limits_{x \in B} B = +\infty$, so there is no number *C* for which it is true that $\forall x \in B: x < C$.

The set *D*= {x > 0| x^(2) > 2} contains elements from the interval $(\sqrt{2}; +\infty)$, hence it is not empty.

This set is bounded below by $\sqrt{2}$, because

$$\forall x \in D: x > \sqrt{2}$$

that follows from the definition of the set *D*.

D is not bounded above because $\sup\limits_{x \in D} D = +\infty$, so there is no number *C* for which it is true that $\forall x \in D: x < C$.

Answer: *B* is not empty. *B* is unbounded. *D* is bounded below by $\sqrt{2}$. *D* is not bounded above.