Answer on Question #69108 – Math – Real Analysis

Question

Evaluate:

lim x→∞ {[2x^(2) +3x-2]^(1/2)}-{[2x^(2) -3x+2]^(1/2)}

Solution

We have

$$\lim_{x\to\infty} \sqrt{2x^2+3x-2} - \sqrt{2x^2-3x+2} =$$

$$= \lim_{x\to\infty} \frac{(\sqrt{2x^2+3x-2} - \sqrt{2x^2-3x+2})(\sqrt{2x^2+3x-2} + \sqrt{2x^2-3x+2})}{\sqrt{2x^2+3x-2} + \sqrt{2x^2-3x+2}} =$$

|We use the formula $(a-b)(a+b) = a^2 - b^2$|

$$= \lim_{x\to\infty} \frac{2x^2+3x-2-(2x^2-3x+2)}{\sqrt{2x^2+3x-2} + \sqrt{2x^2-3x+2}} = \lim_{x\to\infty} \frac{6x-4}{\sqrt{2x^2+3x-2} + \sqrt{2x^2-3x+2}} =$$

$$= \lim_{x\to\infty} \frac{6-\frac{4}{x}}{\sqrt{2+\frac{3}{x}-\frac{2}{x^2}} + \sqrt{2-\frac{3}{x}+\frac{2}{x^2}}} = \frac{6}{2\sqrt{2}} = \frac{3}{\sqrt{2}} = \frac{3\sqrt{2}}{2}.$$

Answer: $\frac{3\sqrt{2}}{2}$.