Answer on Question #69106 – Math – Real Analysis

Question

For the following sequences, find two subsequences which are convergent:

(i) $a_n = n[1 + (-1)^n]$;

Solution

General information about subsequences is here:

http://www-history.mcs.st-and.ac.uk/~john/analysis/Lectures/L9.html

(i) Let us consider the following subsequences:

$a_{2k-1} = (2k-1)[1 + (-1)^{2k-1}] = (2k-1) \cdot [1-1] = 0$, so subsequence a_{2k-1} is convergent, and $\lim_{k\to\infty} a_{2k-1} = 0$.

$a_{4m-1} = (4m-1)[1 + (-1)^{4m-1}] = (4m-1) \cdot [1-1] = 0$, so subsequence a_{4m-1} is convergent, and $\lim_{m\to\infty} a_{4m-1} = 0$.

Answer: $\lim_{k\to\infty} a_{2k-1} = 0$ and $\lim_{m\to\infty} a_{4m-1} = 0$.

Question

For the following sequences, find two subsequences which are convergent:

(ii) $a_n = \sin\frac{\pi n}{3}$.

Solution

(ii) Let us consider the following subsequences:

$a_{3k} = \sin\frac{3k\pi}{3} = \sin \pi k = 0$ (see http://www.bymath.com/studyguide/tri/sec/tri16.htm). So subsequence a_{3k} is convergent, and $\lim_{k\to\infty} a_{3k} = 0$.

$$a_{6m+1} = \sin\frac{6m+1}{3}\pi = \sin\left(2\pi m + \frac{\pi}{3}\right) = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$$

(see https://en.wikipedia.org/wiki/Periodic_function).

So subsequence a_{6m+1} is convergent, and $\lim_{m\to\infty} a_{6m+1} = \frac{\sqrt{3}}{2}$.

Answer: $\lim_{k\to\infty} a_{3k} = 0$ and $\lim_{m\to\infty} a_{6m+1} = \frac{\sqrt{3}}{2}$.