

## Answer on Question #69105 – Math – Real Analysis

### Question

Show that the function  $f$  given by  $f(x) = \frac{1}{(x+2)^3} \forall x \in (-2, 2)$  is continuous but not bounded in  $(-2, 2)$ .

### Proof

Since the function  $f(x) = \frac{1}{(x+2)^3}$  is defined at any point from  $(-2, 2)$ , and  $f$  is elementary function then  $f(x) = \frac{1}{(x+2)^3}$  is continuous in  $(-2, 2)$

(see <https://www.math24.net/continuity-functions/>).

But since  $\lim_{x \rightarrow -2+0} \frac{1}{(x+2)^3} = \infty$  then  $f$  is unbounded in  $(-2, 2)$

(see [https://en.wikipedia.org/wiki/Bounded\\_function](https://en.wikipedia.org/wiki/Bounded_function)). In other words,

$\forall M > \frac{1}{64} \exists x_0 \in (-2, 2): f(x_0) = M$ . Solving the equation  $\frac{1}{(x_0+2)^3} = M$  we get:  $x_0 = \frac{1}{\sqrt[3]{M}} - 2$ , so  $f$  is unbounded in  $(-2, 2)$ .

The statement is completely proved.