Answer on Question #69104 – Math – Real Analysis

Question

Evaluate the following limit if it exists:

$\lim_{x\to 0}\frac{4x^3}{\tan^3 x+\tan x-x}$

Solution

$$\lim_{x\to 0}\frac{4x^3}{\tan^3 x+\tan x-x}=\lim_{x\to 0}\frac{4x^3}{\tan x(\tan^2 x+1)-x}=\lim_{x\to 0}\frac{4x^3}{\frac{\tan x}{\cos^2 x}-x}=[L'Hopital's\ rule]=$$

$$=\lim_{x\to 0}\frac{(4x^3)'}{\left(\frac{\tan x}{\cos^2 x}-x\right)'}=\lim_{x\to 0}\frac{12x^2}{\frac{1}{\cos^4 x}+\frac{2\tan x\cdot\sin x}{\cos^3 x}-1}=[L'Hopital's\ rule]=$$

$$=\lim_{x\to 0}\frac{(12x^2)'}{\left(\frac{1}{\cos^4 x}+\frac{2\tan^2 x}{\cos^2 x}-1\right)'}=\lim_{x\to 0}\frac{24x}{\frac{4\sin x}{\cos^5 x}+\frac{4\tan x}{\cos^4 x}+\frac{4\tan^2 x\cdot\sin x}{\cos^3 x}}=[L'Hopital's\ rule]$$
$$=$$

$$=\lim_{x\to 0}\frac{(24x)'}{\left(\frac{8\tan x}{\cos^4 x}+\frac{4\tan^3 x}{\cos^2 x}\right)'}=\lim_{x\to 0}\frac{24}{\frac{8}{\cos^6 x}+\frac{32\tan x\cdot\sin x}{\cos^5 x}+\frac{12\tan^2 x}{\cos^4 x}+\frac{4\tan^3 x\cdot\sin x}{\cos^3 x}}$$
$$=\frac{24}{8+0+0+0}=3$$

Answer: 3.